



New York State Technology Enterprise Corporation

and



Syracuse Research Corporation

present a

Technical Report for

**Generation of the National 700-MHz
Public Safety Pool Allotments
(Narrowband General Use Channel Set)
Documentation of Methodology and Results**

Submitted to:

Mr. David Funk

Deputy Manager, Communications Technology Outreach
National Law Enforcement and Corrections Technology Center

and the

The National Public Safety Telecommunications Council

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Version 1

Generation of the National 700-MHz Public Safety Pool Allocations (Narrowband General Use Channel Set) — Documentation of Methodology and Results



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1. EXECUTIVE SUMMARY

The 700-MHz spectrum was not available for use by land-mobile radio operations until recently. This new availability offers many exciting possibilities for creating new paradigms in the way this spectrum is allotted and used. In particular, the use of more detailed models within the pre-allotment and regulatory realms can allow for a higher level of spectral efficiency than was previously achievable. The NYSTEC/SRC Team, utilizing its spectrum-management expertise, has been given the opportunity to generate pre-allotments pools for the general-use (narrowband) Public Safety portion of this band. After several months of effort, the Team has successfully defined these allotments. This paper provides insight into the process of generating these allotments, as well as documents the methodologies used to generate this initial 700-MHz allotment pool. A summary of methodologies utilized follows.

- Used population and population-density characteristics in evaluating capacity needs. Employed PSWAC-like capacity requirement models to introduce increased accuracy in the modeling process.
- Used terrain data for service-area evaluation and interference prediction. This enabled greater accuracy in the pre-allotment process, and resulted in efficient reuse of the spectrum.
- Used contour intersections to evaluate the validity of pre-allotment channel sets. Built upon the Team’s past experience in developing quasi-optimal spectral allotment solutions.
- Pre-allotted “pool” channels in aggregate 25-kHz blocks. Provided a minimum¹ of five blocks per allotted (county-like) area — four for voice, and one for data. Allotted additional spectrum based upon projected need (normalized by the spectrum available) and local availability.
- Allotted all 700-MHz Regions, which included all fifty states plus Puerto Rico and the Virgin Islands.
- No attempt was made to work around either U.S. or International broadcast-television services. Many of these station assignments are either temporary or subject to change, and working around them would have resulted in allotment inefficiencies.

Although doing so would be outside the scope of this effort, the NYSTEC/SRC Team could, at a later time, re-run the program in order to update it with additional Regional Planning Committee allotment application data, and could revise and re-pack the “pool” pre-allotments within those regions accordingly. Such periodic maintenance would ensure that the 700-MHz spectrum would remain an efficiently deployed resource.

¹ At least five allotments were provided for all counties, except for those within the Puerto-Rico (Region 47) and Virgin Islands (Region 48) Regions.

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2. INTRODUCTION

This paper provides an insight into the process and documents the methodology used to generate the initial 700-MHz allotment pool.

Until recently, the 700-MHz spectrum was not available for use by land-mobile radio operations. This new availability offers many exciting possibilities for creating new paradigms in the way that it is allotted and used. In particular, the use of more detailed models within the pre-allotment and regulatory realms allow for a higher level of spectral efficiency than was previously achievable.

Regulatory and Rulemaking procedures for the 700-MHz Public Safety Narrowband spectrum are nearly complete. Once the regulatory and rulemaking processes close, many areas of the country will be able to make immediate use of the 700-MHz spectrum (pending equipment availability). Furthermore, most statewide reserve allocations of this spectrum are already licensed. Because of these factors, there was a genuine need for pre-allotment of the spectrum, especially for frequency coordination and Regional Planning purposes. Pre-allotment produces “pools” of channels that may be used in a given area. As actual application data is received from Regional Planning Committees, the process can be run again to re-optimize the “pool” allotments that would remain available within a Planning Region.

2.1 The Need for Pre-allotment

NPSTC has made available to all authorized frequency coordinators a pre-allotment database for the new 700-MHz narrowband public-safety spectrum. In order to maximize the utility of NPSTC’s pre-coordination database and to enable its use within frequency coordination and regional planning, it was important to completely populate the database as soon as possible. In order to accomplish this, it was necessary to perform the allotments on a national basis.

This database is now complete and, therefore, nearly ready to be populated with the initial “pool” allotments. At one time it was anticipated that the allotments would be provided over time on a regional basis — and with input required from approximately 55 individual regional planning committees. As a result of the NYSTEC/SRC effort, the allotments have developed all at once on a national basis, and without the need for massive collaborative efforts from the individual regional committees — many of which have not yet formed.

2.2 Pre-Allotment Regions and Boundaries

The geographical structure of the 700-MHz Regional Planning Committees (RPCs) is loosely based upon state borders, as is shown in Figure 1 (continental U.S.) and Figure 2 (all Regions). However, note that some states (e.g. Texas) are broken into multiple regions, and some Regions (e.g. Region 19) span multiple States.

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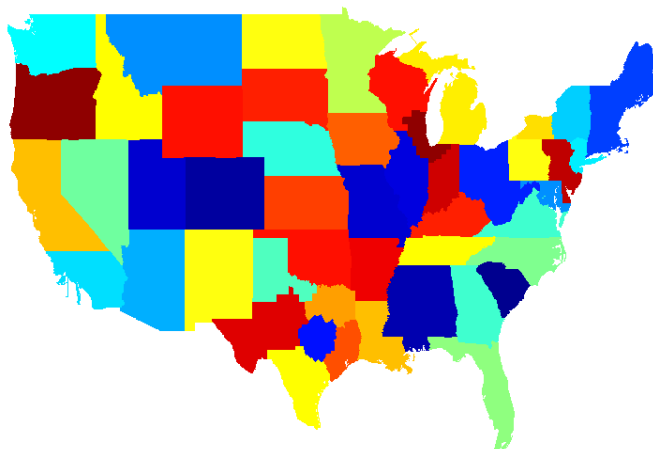


Figure 1, Continental U.S. 700-MHz Regions



Figure 2, All 700-MHz Regions

Site-specific parameters are not available during the pre-allotment process. However, the spectrum must be allotted based upon some pre-defined type of bounded area. The obvious choice (and with the precedent set when previous allotments were done for 800-MHz spectrum) was to allot the spectrum based upon “county-type boundaries.” “County-type boundaries” means the county boundaries plus municipalities and cities that do not fall within a county and which will be treated as their own individual allocable areas. The 700-MHz narrowband spectrum has been pre-allotted according to these boundaries, with *most* public-safety usage falling naturally into these subdivisions. A map of the county-type divisions that were used for this effort is shown in Figure 3, again for the continental U.S.

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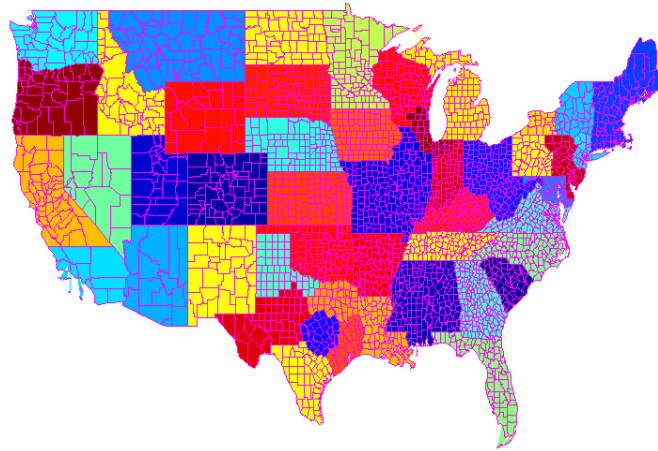


Figure 3, Continental U.S. 700-MHz Regions, with County Boundaries

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3. OVERVIEW OF METHODOLOGY

The methodologies that were used for allocating the pool spectrum in this effort were much more detailed than those previously available for the NPSPAC allocations. This should allow for a higher level of spectral efficiency than has been possible in past efforts of this nature, as well as more accurate interference characterization and better estimation of relative capacity needs. This section provides an overview of the parameters used and the methodologies employed.

3.1 Spectral Needs Assessment

Based upon discussions with the NPSTC database subcommittee, it was decided that each indicated county/area would receive some minimum allotment (e.g., three 25-kHz channel pairs for voice and one 25-kHz channel pair for data), regardless of aggregate capacity needs. Beyond this, the pre-allotment process would provide additional spectrum based upon some measure of individual capacity needs. In the past, this additional capacity assessment was based solely on population. This past approach has been modified to more accurately capture the relative capacity need of each county pool recipient.

In the NYSTEC/SRC Team's analysis of public-safety capacity needs within New York, it was found that these needs varied tremendously across the State. It was clear that there was a strong correlation between population and public-safety capacity needs. However, it was also found that, by only considering county populations, a large number of public-safety and public service users were not accurately represented in the rural areas. This is illustrated in Figure 4.

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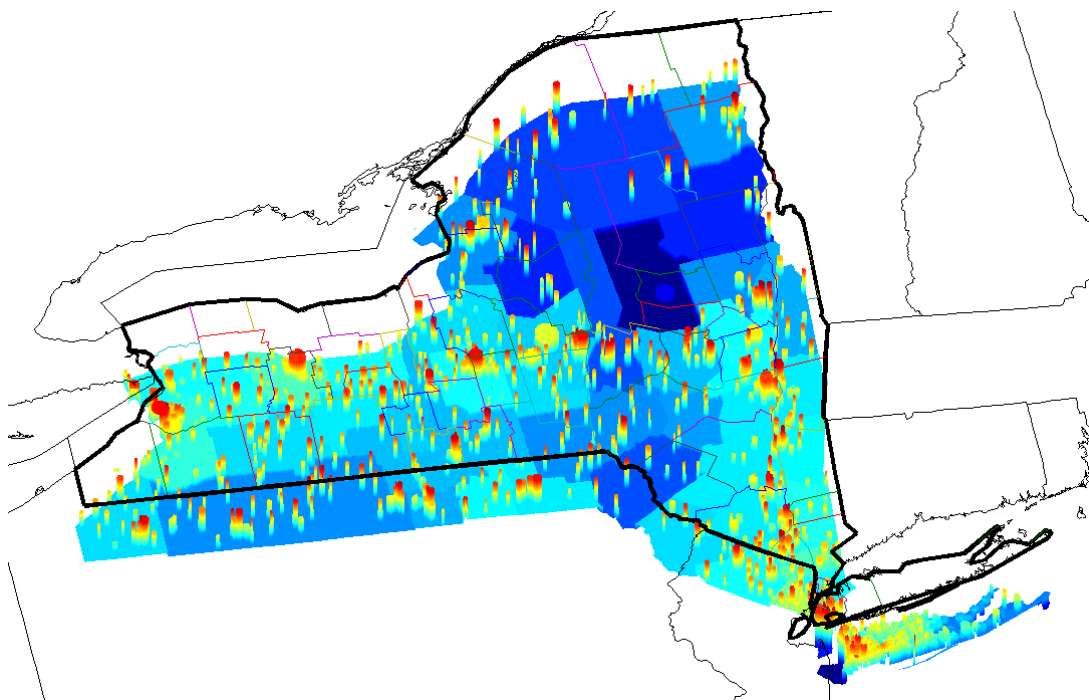


Figure 4, Traffic Gradient across a State (New York)

The NYSTEC/SRC Team utilized an approach similar to the PSWAC² approach, in which both population and population density were used to predict the total number of public-safety users within a specific area to be allotted spectrum. The most recent population data available, the 2000 census, was used to provide the model baseline. However, PSWAC’s models needed to be modified — since the original models incorporated little data from rural areas.

Once public-safety and public service user populations were projected for a given area, they were used to distribute the spectrum pre-allotments, normalized by the total amount of available spectrum (with reuse), and by the total national public-safety user projections.

3.2 Service Area Definition and Interference Evaluation

It was clear that accurate modeling of coverage and interference effects allows for tighter site/frequency “packing” and greater spectral efficiency. Again, since this frequency band is a new allocation, the Team was able to utilize more accurate methods of assessing these effects during the pre-allotment stages of spectrum planning and plan development. The NYSTEC/SRC Team has experience in developing innovative techniques for spectral assignment processes, and continues to work with Telecommunications Industry Association (TIA) TR-8.18 working

² FINAL REPORT OF THE PUBLIC SAFETY WIRELESS ADVISORY COMMITTEE TO THE FEDERAL COMMUNICATIONS COMMISSION (Reed E. Hundt Chairman) AND THE NATIONAL TELECOMMUNICATIONS AND INFORMATION ADMINISTRATION (Larry Irving, Assistant Secretary of Commerce for Communications and Information), September 11, 1996.

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groups in developing the next generation of coverage- and interference-assessment methodologies.

For the 700-MHz pre-allotments, the service area/contour for each of the counties was represented by a bounding polygon that extended beyond the county border by 3 to 5 miles. This actual distance from the county border was individually based upon population-density metrics (NCC recommendations call for 3 miles for rural areas and 5 miles for urban areas).

There were several possibilities for generating the interference contour(s), all utilizing some measures of local terrain characteristics. It is clear that utilization of terrain features enables a much more accurate representation of signal propagation and interference prediction, especially when compared to simple “rule-of-thumb” reuse distances.

With no site-specific information available, there were many ways to predict frequency reuse parameters and produce the interference contours. Several methods were investigated as part of this effort, and the NYSTEC/SRC Team chose the method that appears to best model the actual physical interference mechanisms.

3.3 Allotment Approach

NYSTEC and SRC also have experience in generating spectrally efficient frequency assignment methodologies — as evidenced by past work generating spectrum plans for a statewide wireless network, and by generating and proposing alternative Digital Television Transition plans for Canada³. This experience was leveraged toward the optimization of certain parameters of the pre-allotment pool.

3.4 Basic Allotment Approach

The spectrum-allotment approach was based upon the non-intersection of contours — an approach familiar to regulators and frequency coordinators alike. Specifically, it applied rules within the allotment process to specify that service and interference contours for co-channel frequency allotments cannot intersect. In addition to this, it did not allow adjacent-channel interference contours to intersect service contours on an adjacent-channel examination.⁴ The program spreads out into the massive search space, so that, if not enough spectrum were available to meet the recommended levels of any given county, it spread the load out over all counties involved within the allotment process. This ensured that every county reached a similar level of capacity — relative to its projected needs.

This process provided the ability to pack the spectrum geographically to a very large degree. Note that, while not part of this proposed scope of work, the NYSTEC/SRC Team could also provide periodic re-packing of this spectrum, once site-specific licenses are issued and more

³ These Canadian plans would completely eliminate the need for 700-MHz DTV allotments, and essentially align 700-MHz spectrum on both sides of the U.S./Canadian border.

⁴ TIA’s recommendations of 60-dBu contour values for adjacent-channel interference (based upon 65-dB ACCPR into a 6.0 kHz signal) essentially drove this into a service-service adjacent channel requirement.

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detailed models can be applied⁵. Note that, *when* site-specific parameters are available, it is important to populate the database with contours that represent coverage and interference parameters as accurately as possible. For this, a tile-based contouring method is recommended.

The pre-allotment process also accounted for realistically achievable antenna system and multi-coupler spacing. For this reason, all-individual pre-allotment channel sets have an internal separation of no less than 250 kHz.

3.4.1 Allotted Bandwidth

One very important parameter of the pre-allotment process was the bandwidth of the pre-allotted voice and data channels. This has proved to be a strongly debated topic of discussion.

The potential for many diverse technologies within the same spectrum is troublesome with regard to determining the smallest building blocks to allot. It was clear to see that the spectrum may be allotted in 6.25-kHz portions (allowing the use of future FDMA technologies), or 12.5-kHz “bundles” (allowing the use of current FDMA and future TDMA technologies), or 25-kHz “blocks” (allowing the use of 25-kHz TDMA technologies). The inherent problem was that allotting anything smaller than 25-kHz blocks would preclude the future use of 25-kHz technologies on the pre-allotted channel sets. Presently, no U.S. 25-kHz TDMA technology product is available for operation in this band, although FCC Rules allow such operation.

NPSTC and TIA have previously recommended that 25-kHz blocks be pre-allotted for both voice and data applications. At the May 2001 NCC meeting, it was proposed that three (3) 25-kHz voice channels and one (1) 25-kHz data channel be the minimum default allotments in the absence of actual specific applications for channel allotment. This would permit different technologies to be implemented using 6.25-, 12.5-, or 25-kHz channel widths at some future date. Therefore, pre-allotments have been generated based upon aggregating 25-kHz blocks of spectrum.

3.4.2 Geographic Boundaries

NPSTC has previously recommended that the pre-allotments be performed only along the borders of each region. After discussions with the NYSTEC/SRC Team, it was seen that better spectral efficiency could result from allotting all areas of all regions during the pre-allotment process. Pre-allotment of all areas, even within regions, can also result in significantly faster availability of channels to an applicant, since the regional planning process has already taken place. Otherwise, one might have to wait for a regional planning process to follow an application.

NYSTEC/SRC generated the pre-allotments throughout all of the regions, but modifications to allotments outside of the border areas should be allowed without restriction by individual regional planning committees, and without the need for inter-regional coordination. However, if

⁵ This periodic maintenance will ensure that the 700 MHz spectrum remains an efficiently deployed resource.

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such changes result in interference impacts to any adjacent region, inter-regional concurrence must remain a mandatory requirement.

3.4.3 Treatment of Television Services

There were many additional constraints that could have been imposed upon the pre-allotment process; most are based upon the existence of current and future television broadcast services within the 700-MHz band. These would include incumbent U.S. analog stations as well as U.S. digital allotments that occur in certain areas of the nation. A disturbing problem is the uncertainty related to international broadcast services (in particular Canada and Mexico) that may claim protection from, and cause interference to, U.S. operations within the spectrum.

While it is possible to alter the allotment process to take all of these broadcast services into account, the final result will not provide the same spectral efficiency that would otherwise be possible. It is also possible that consideration of all of the stations may over-constrain the problem, generating inefficient results for no valid reason.

The actual selection of allotment criteria and stations to consider during the allotment process depended on many factors — among them U.S. 700-MHz spectrum availability; the DTV transition timelines of the U.S., Mexico, and Canada; and international negotiations and treaties. However, for the purposes of this proposal, NYSTEC/SRC gave no consideration to allotting spectrum based upon broadcast television services emanating from within the U.S. or abroad.

It should be noted that an effective mechanism for mitigating border-area spectrum shortage is a natural by-product of the allotment-generation process. In general, border areas receive more allotments due to the reduced-interference constraints that they experience. Since the mandatory frequency separation within each received pool allotment essentially spaces the received spectrum block over the TV channel range (i.e. 63/68 and 64/69), the border areas are better able to deal with the spectrum blockage that they may experience due to international services.

3.4.4 Consideration of Existing Regional Plans

Due to the lack of availability of existing Regional Plan Allotments, these were not included within the pre-allotment pool that was generated under this effort, but would be part of any periodic re-packing or band-maintenance effort.

3.5 Methodology Summary

In order to maximize the utility of NPSTC's 700-MHz public safety pre-coordination database, and to effectuate its use for regional planning and frequency coordination in a multiple vendor environment, it was imperative to populate the pre-coordination database with pool allotments. In order to accomplish this with optimal spectral efficiency, it was necessary to perform the pre-allotments on a national basis, and to utilize accurate models and spectral assignment strategies.

A summary of methodologies utilized follows:

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- Use population and population-density characteristics in evaluating capacity needs. Employ PSWAC-like capacity requirement models to introduce increased accuracy in the modeling process.
- Use terrain data for service-area evaluation and interference prediction. This will allow greater accuracy in the pre-allotment process, and will result in more efficient reuse of the spectrum.
- Use contour intersections to evaluate the validity of pre-allotment channel sets. Build upon past experience in developing quasi-optimal spectral allotment solutions.
- Pre-allot “pool” channels in aggregate 25-kHz blocks. Allow a minimum⁶ of four blocks per allotted (county-like) area — three for voice, and one for data. Allot additional spectrum based upon projected need (normalized by the spectrum available), and local availability.
- Allot all areas of the U.S. as listed in Appendix A, which includes all fifty states and Puerto Rico.
- When considering allotable spectrum blocks, make no attempt to work around either U.S. or International broadcast-television services. Many of these station assignments are either temporary, or subject to change, and working around them would have resulted in allotment inefficiencies.

Although outside the scope of this effort, the NYSTEC/SRC Team could, at a later time, re-run the program in order to update it with additional Regional Planning Committee allotment application data, and revise and re-pack the “pool” pre-allotments within those regions accordingly. This periodic maintenance would ensure that the 700-MHz spectrum remains an efficiently deployed resource.

⁶ At least five allotments were provided for all counties, except for those within the Puerto-Rico (Region 47) and Virgin Islands (Region 48) Regions.

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4. GENERATION OF CAPACITY MODELS

In order to distribute the pool allotments so as to meet the requirements of the individual counties, the capacity needs of the counties themselves needed to be quantified. In the past, this metric was based purely upon total population within the counties. Within this effort, more-detailed models were utilized, ones that better capture the characteristics of communications traffic load on Public Safety and Public Service networks, as well as the associated capacity requirements. These models are described and documented within this section.

4.1 Existing PSWAC Models

The models developed under this effort leverage the massive efforts of the Public Safety Wireless Advisory Committee (PSWAC), and is based upon the work documented in the PSWAC Final Report⁷. In this Report (particularly within portions by the Operational Requirements and the Spectrum Requirements Subcommittees), the traffic loading of public-safety and public-service personnel was examined, and analytical models were developed in order to estimate the spectrum requirements of Public Safety/Service through the year 2010. These models (and most traffic models) required the following parameters:

- Number of personnel per service (e.g., Fire, Police, EMS),
- Fraction of voice and data utilization within each service,
- Average and peak voice traffic loading per unit for each service, and
- Average and peak data traffic loading per unit for each service.

Through data collection, analyses, and debate between industry experts, preliminary values and/or models for these parameters were developed. Parameters for service penetration and per-unit traffic loading were presented⁸, as were curves⁹ that modeled the population of each service category.

Table 1 presents the per-unit voice and data traffic levels for Law Enforcement, Fire, Emergency Medical, and Local Government services. These values encompass the PSWAC parameters, accounting for the voice, data, and status message types — the types expected to be carried by the 700-MHz Public Safety General Use narrow band channels. Rather than being presented in their original traffic units (Erlangs), these are translated to units more easily understandable to the end users — such as airtime per user, and data transfer per user. These values were utilized within the new models developed under this effort.

⁷ FINAL REPORT OF THE PUBLIC SAFETY WIRELESS ADVISORY COMMITTEE TO THE FEDERAL COMMUNICATIONS COMMISSION (Reed E. Hundt Chairman) AND THE NATIONAL TELECOMMUNICATIONS AND INFORMATION ADMINISTRATION (Larry Irving, Assistant Secretary of Commerce for Communications and Information), September 11, 1996.

⁸Appendix D and Appendix G – Spectrum Requirements Subcommittee (SRSC) Final Report.

⁹Appendix A - Operational Requirements Subcommittee (ORSC) Final Report.

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Table 1, Voice and Data Loading for the Capacity Model

Agency Type	On-System Voice Airtime per Active User per Hour (minutes)		Data Transfer per Active User per Hour (KB@3kbs)	
	Peak	Average	Peak	Average
Government	2.6	0.7	11.7	2.9
Police	3.3	0.8	11.7	2.9
Fire	2.9	0.7	11.7	2.9
EMS	2.9	0.7	11.7	2.9

Derived from Appendix G – Spectrum Requirements Subcommittee Final Report (also Appendix D), Note that the second column in the Appendix G tables should actually be labeled “Peak ERL/User”, as opposed to “Avg ERL/User”, in order to be consistent.

The PSWAC service population models predicted each service’s user population as a percentage of total population, and as a function of population density. Approximate values for these models are presented within Figure 5. These are discussed further in the next section.

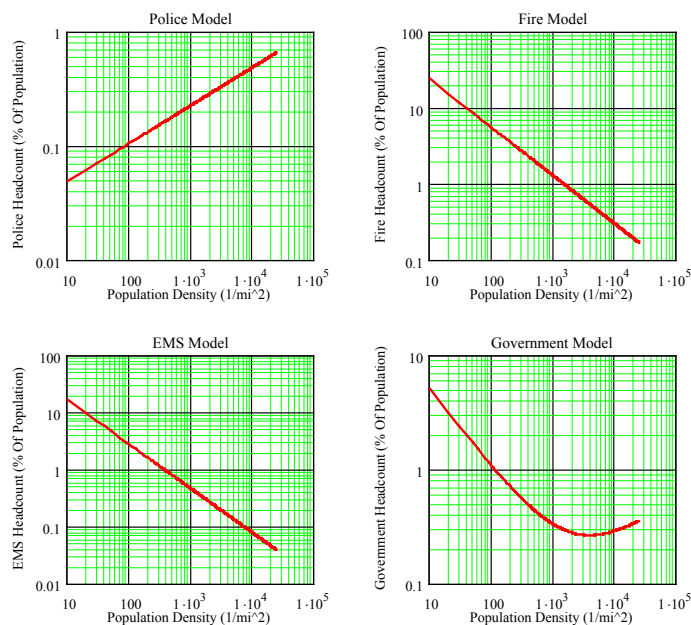


Figure 5, Original PSWAC User Population Models

4.2 Augmentation of PSWAC Models

There is no disputing that the work of the PSWAC stands as *the* standard for Public Safety requirement definition, and that the material contained therein embodies the consensus of an unparalleled group of experts. However, some of the models and concepts developed by the

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PSWAC do require some modification and augmentation in order to apply them to modeling the capacity needs of all U.S. counties.

The main modification necessary to the original PSWAC models is an extension of the range of the User Population models, and a shift in the values of these models at the lower edge of their usable ranges. There are two reasons for this: 1) the sizes of the data sets used in the original curve fitting were limited, and 2) there was little or no data utilized at low-population-density levels. For the Police model, there was no data used for population-density values less than $100/\text{mi}^2$, and for the Fire and EMS services there was no data for levels less than $1000/\text{mi}^2$. Within this effort, the Team needed to estimate a measure of capacity need for all Counties, with an average population density of $270/\text{mi}^2$ and a median density of $44/\text{mi}^2$. Only 30% of the population-density values that need to be considered fall within the range of the PSWAC Law Enforcement values, and only 5% within the range PSWAC used for Fire and EMS personnel. Furthermore, it is clear that, without a breakpoint in these models, the Fire and EMS models would predict infinite personnel as the population density approaches zero (the police models would predict zero police personnel under the same circumstances). In order to modify these models, additional data was collected and analyzed. This data was then used to augment the usable range of the PSWAC models.

Figure 6 and Figure 7 present additional data that was used to extend the range of the PSWAC Law Enforcement model. In Figure 6, FBI data is compared to data collected from New York State (NYS) Counties and Municipalities. While the FBI data is a time series, it is seen to converge to the same slope as the NYS data, i.e. 280 sworn officers per 100,000 population. From this, the Team set the breakpoint of the PSWAC Law Enforcement model so that the model is unchanged for values greater than $1950/\text{mi}^2$ and flattens out at 0.28% for population-density values lower than $1950/\text{mi}^2$. The resulting model is shown in Figure 8, in which the additional data set (blue points) is superimposed on the Final model (solid red line).

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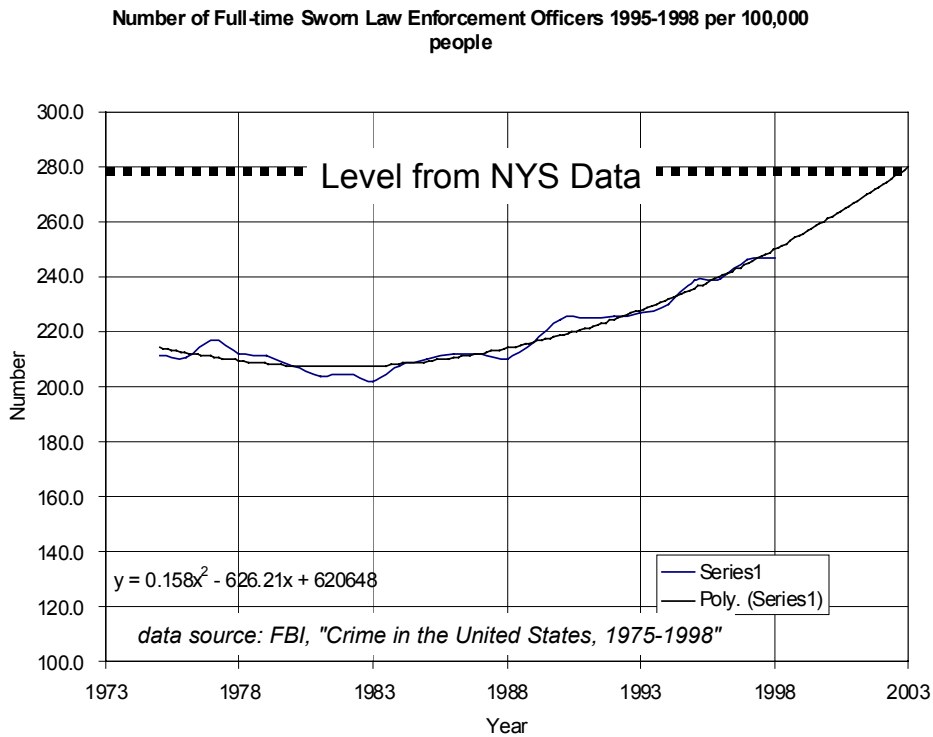


Figure 6, Additional Law Enforcement Data (FBI) for Model Augmentation

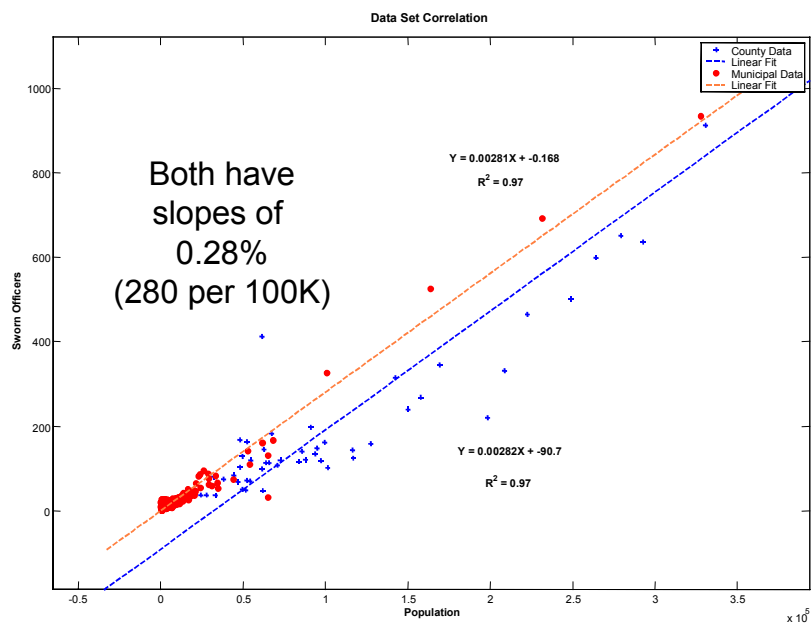


Figure 7, Additional Law Enforcement Data (NYS) for Model Augmentation

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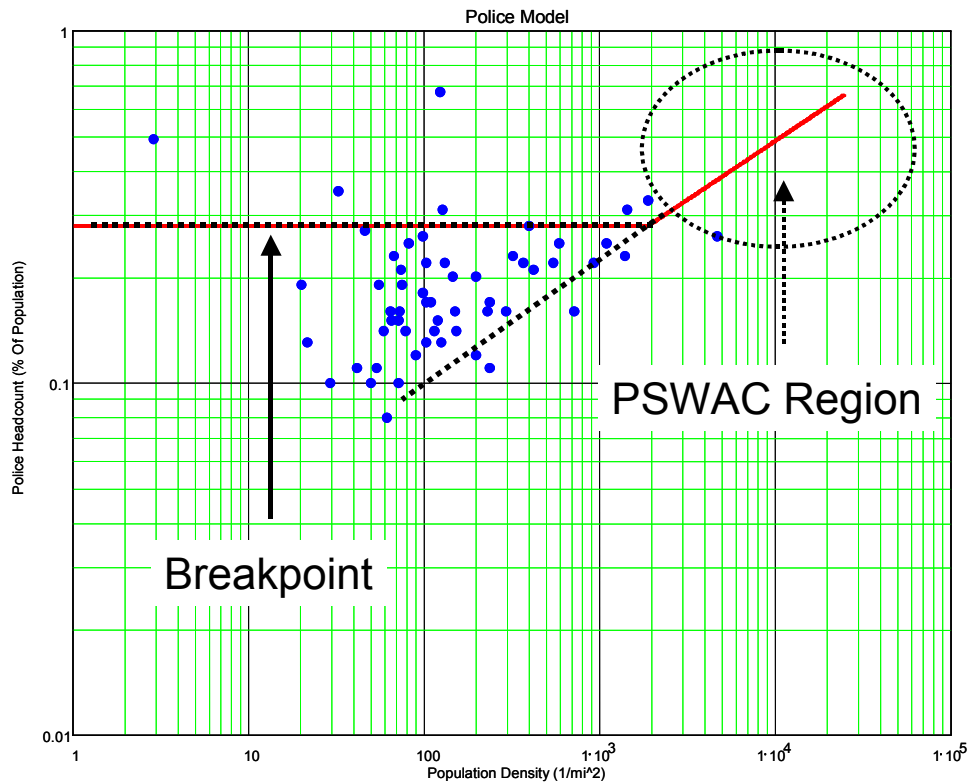


Figure 8, Modified PSWAC Law Enforcement Population Model

Figure 9 presents additional data from New York State that was used to extend the range of the PSWAC Fire Services model. From this, the Team set the breakpoint of the PSWAC Fire Services model such that the model is unchanged for values greater than $4920/\text{mi}^2$, with a slope change to match the NYS data for population-density values lower than $4920/\text{mi}^2$. The resulting model is shown in Figure 10. Not apparent from this figure is an additional breakpoint that does not allow the predicted service population to exceed 5%. This was required so that extremely low population-density areas would still be within the range of the model. In this figure, the additional data set (blue points) is superimposed on the Final model (solid red line).

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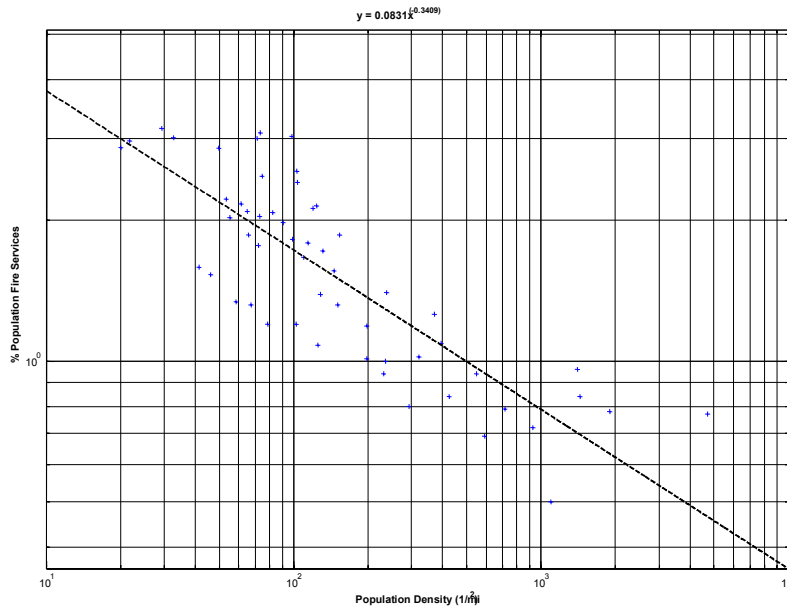


Figure 9, Additional Fire Personnel Data for New York Counties

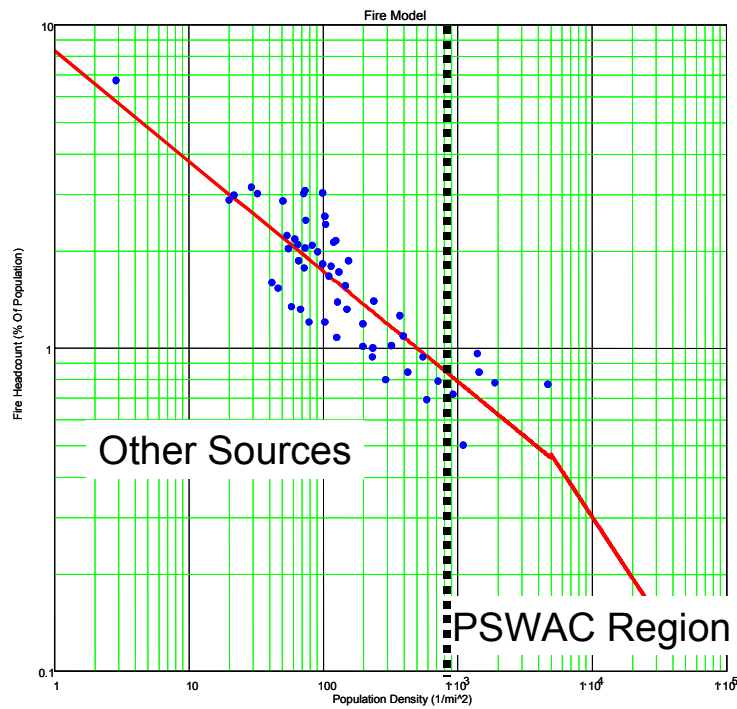


Figure 10, Modified PSWAC Fire Services Population Model

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Figure 11 presents the modified PSWAC EMS Services model. Using additional data¹⁰, the Team set a breakpoint of the EMS model values lower than 1250/mi², and left values higher than 1250/mi² unchanged. As with the other figures, the additional data set (blue points) is superimposed on the Final model (solid red line).

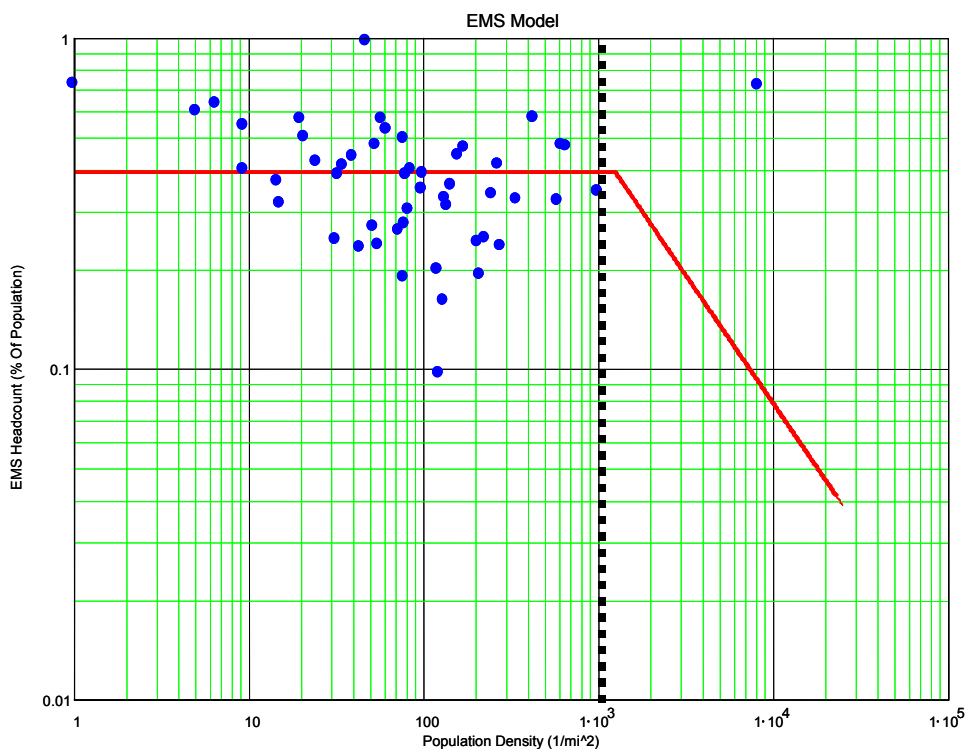


Figure 11, Modified PSWAC EMS Population Model

4.3 Final Results

Using 2002 U.S. Census data, service populations were generated from the models documented in subsections 4.1 and 4.2. The overall results of this are shown in Figure 12, in which they are seen to match the models developed in 4.2, and in Figure 13, in which the strong correlation to overall population levels is emphasized. Note from Figure 12, that these models show that size of the service population in rural areas is in order from highest to lowest: Fire, Government, EMS, and Police. In urban areas these levels shift to (again, from highest to lowest): Police, Government, Fire, and EMS. This is consistent with standard reasoning; for example, the Police percentage increases from rural to urban areas due to the increased crime rate. The Fire and EMS percentages decrease from rural to urban, as volunteer services become replaced by full time professional services.

¹⁰ EMS Magazine, 2002 Emergency Medical Services survey, A complete collection of EMS data from all 50 U.S. states, the District of Columbia, the U.S. territory of Puerto Rico, the U.S. Virgin Islands, and the Canadian provinces. <http://www.emsmagazine.com/SURVEY/index.html>

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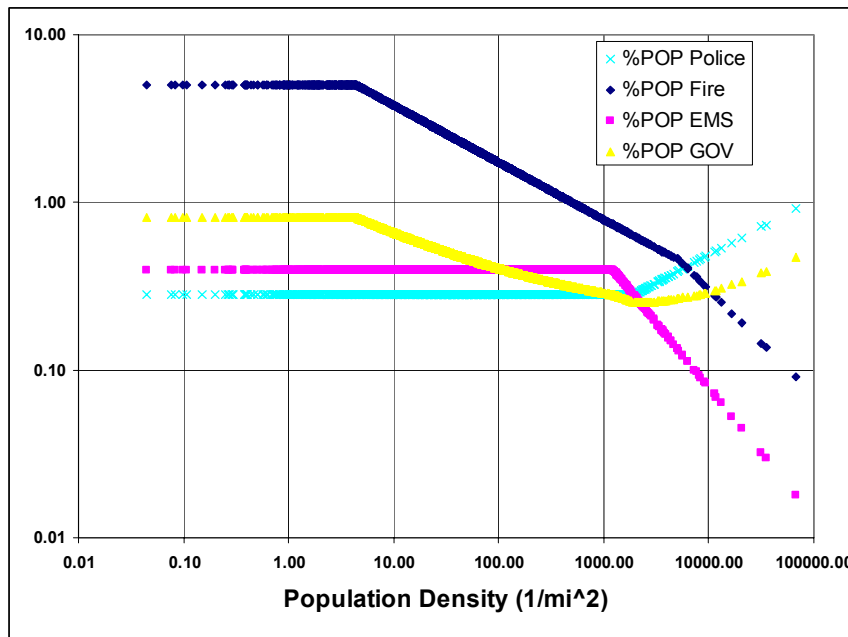


Figure 12, Final % Service Population Models vs. Population Density

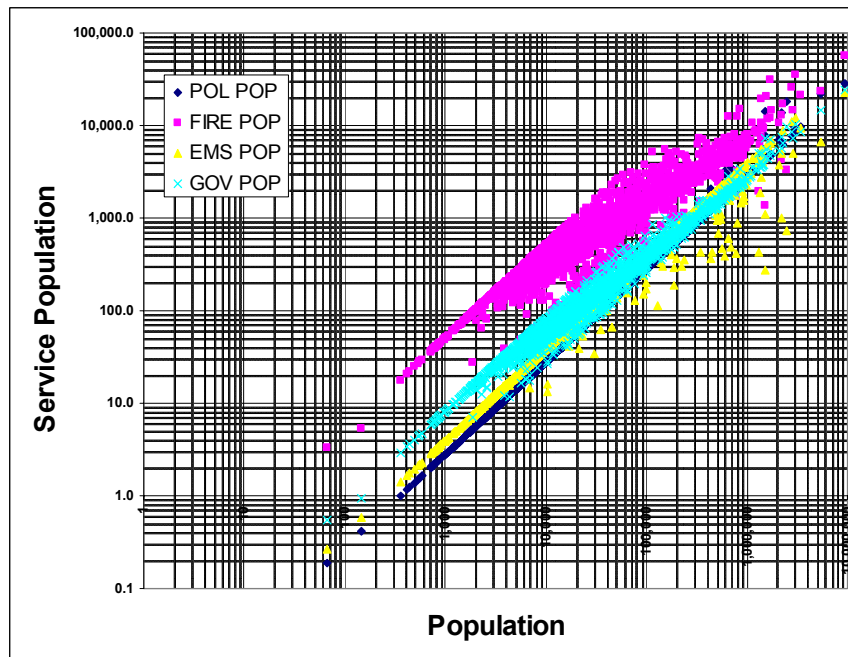


Figure 13, Final Service Population Models vs. Population

For each County j , these service populations $POP_{i,j}$ were then translated into traffic (Erlang) loading through the consideration of:

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- Voice and Data Service Penetrations, Pv_i, Pd_i – The expected fraction of the i^{th} user group requiring a particular communications process.
- Operational Time Schedules, S_i – The expected fraction of the i^{th} user group that is active during the typical busy-hour of the day.
- Average per Unit/User Loading, Av_i, Ad_i – The expected per-user voice and data traffic loading for of the i^{th} user group averaged over the course of a day
- Peak per-Unit/User Loading Bv_i, Bd_i – The expected per-user voice and traffic loading for the i^{th} user group during the busiest one-hour period of a day.

This then gives the normalized capacity requirement of each county area, C_j as:

$$C_j = \frac{\sum_i POP_{i,j} \cdot S_i (Pv_i \cdot Bv_i + Pd_i \cdot Bd_i)}{\left\| \sum_i POP_{i,j} \cdot S_i (Pv_i \cdot Bv_i + Pd_i \cdot Bd_i) \right\|_{\infty}}$$

where the denominator is the ∞ -norm over the numerator vector, or the maximum capacity requirement that is obtained by the model. PSWAC recommendations for these values were followed, augmented by other data as necessary. In particular, shift levels of 50% (Police, Local Government) and 60% (Fire and EMS) were applied. These shift levels were arrived at through past interviews and discussions with end users and user groups. The final results were normalized so that the maximum capacity requirement was unity. The overall distribution of the resulting normalized capacity requirements is shown in Figure 14, with the geographic distribution over the Continental U.S. shown in Figure 15. These per-county normalized Erlang requirements were normalized to give relative metric of each county’s capacity needs. When employing these within the generation of the pool allotments, no conversion to channel requirements (i.e. Erlang-C) was made.

A full list of the Counties, with population, area, and pre-normalized capacity requirement, is included as Appendix B, “Population, Area, and Capacity Model Data.”

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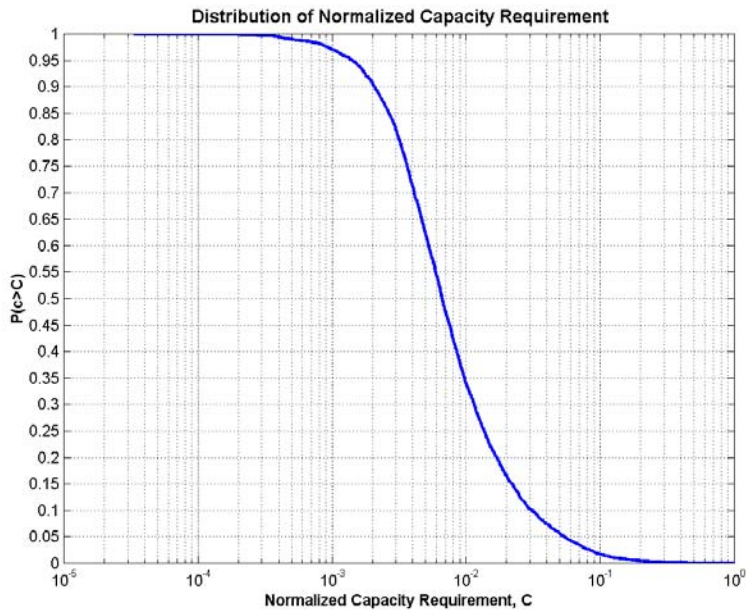


Figure 14, Distribution of Normalized Capacity Requirements

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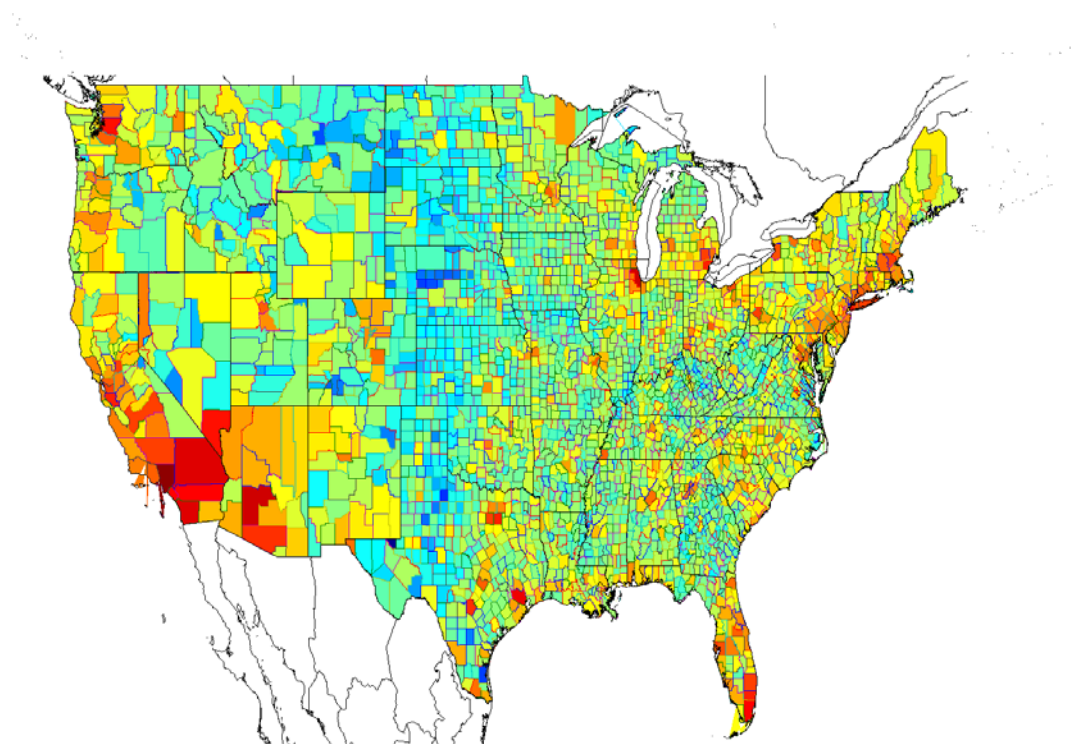


Figure 15, Final Capacity Model - Continental U.S.

Figure 15 shows that the final capacity requirement model is highly variable across the country, and has “hot-spots” in the highly populated areas (as would be expected), as well as “cold-spots” in sparsely populated areas. The gradient and other characteristics of the capacity requirement are easier to discern in a more localized view, such as presented in Figure 16 for the Region 25/St. Louis (MO). In this Figure, the top six “hot-spots” (in order of highest capacity requirement downward) are: St. Louis County, Jackson County, St. Charles County, Green County, Jefferson County, and St. Louis City. Note that, although these are the six most highly populated county areas within the Region, the capacity model does not strictly follow only population. Despite having lower population levels, St. Charles, Green, and Jefferson Counties all have slightly higher capacity requirements than does St. Louis City. This is an intentional characteristic of the model(s) and reflects the concepts illustrated in the PSWAC Report.

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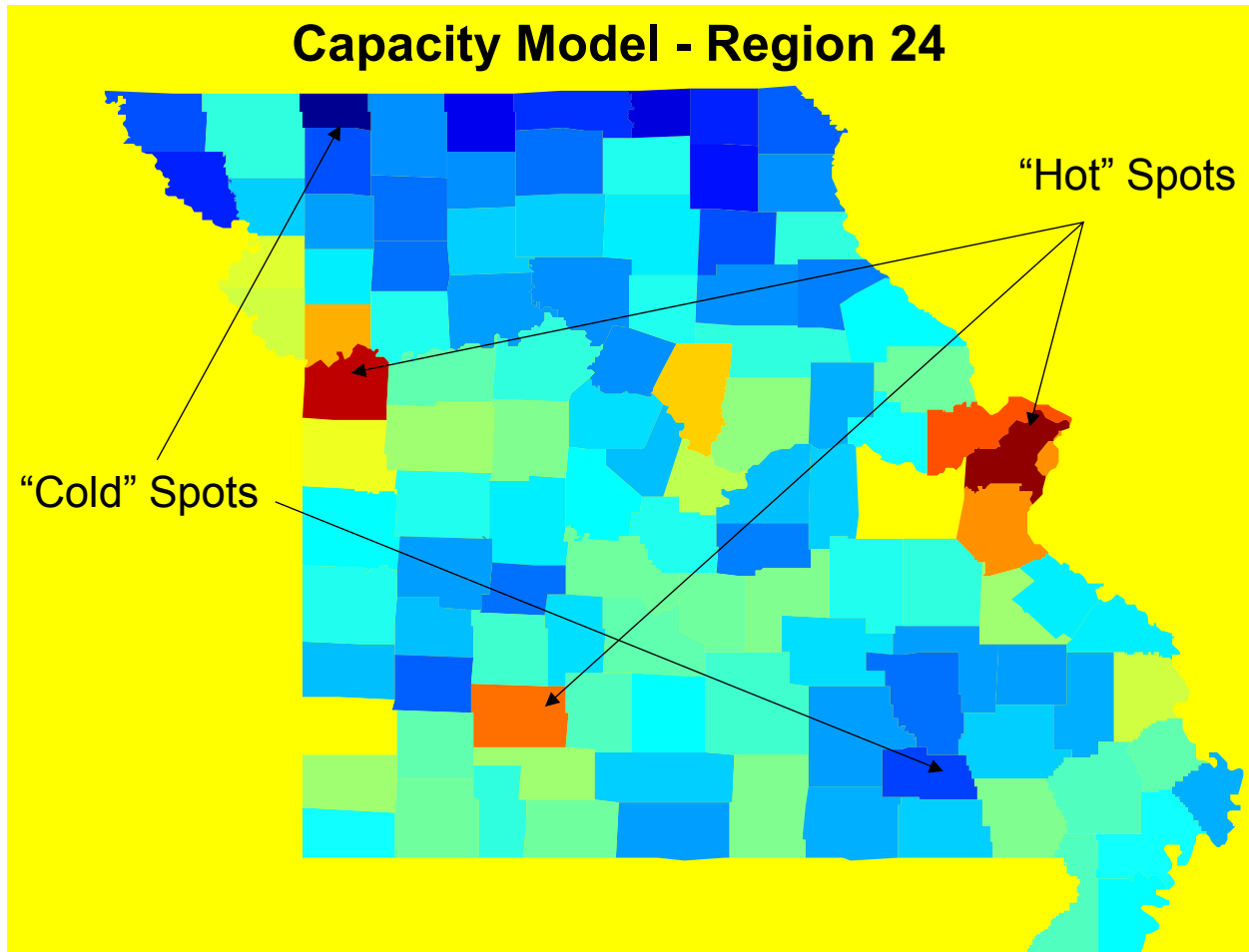


Figure 16, Final Capacity Model - Detail of Region 24

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5. GENERATION OF SERVICE AND INTERFERENCE CONTOURS

In order to determine which of the county areas would or would not interfere with each other, two contours were generated for each county. The first, the service contour, encloses the region in which that county's public-safety radios should be able to operate. The second, the interference contour, describes the region in which that county's public safety communications system may interfere with any other radios attempting to operate on the same channel. This section describes the characteristics, usage, and generation of these contours.

It should be noted that the methodologies described herein have been discussed in Public Forum¹¹ and have also been provided to select members of the Public Safety Frequency Coordination and Regional Planning Communities. In these discussions, the methodologies were deemed sound and to represent well the interference potential that is seen in real-life implementation and coordination.

5.1 Service Contours

The service contour for each county encloses the region in which that county's public safety radios should be able to operate. This contour is based on both the shape of the county and the population density of the county. It creates a buffer region three or five miles outside of the county border, based upon population density. If the county's population density is in the highest quartile for the nation, the five-mile buffer defines the service contour; otherwise, the three-mile buffer is used. These values were based upon the recommendations of the National Coordination Committee's Implementation Subcommittee¹², and are intended to indirectly address the specific portable coverage needs for urban (5-miles) and rural (3-miles) areas by allowing higher powers within the County boundary.

In Figure 17, the black shape is the border of a typical county, Henry County, Kentucky. The blue is Henry County's service contour. The county's public-safety radios are only routinely expected to operate within the black outline, but a three-mile buffer is included in the service contour as previously discussed. This also helps account for spillover radiation that is especially difficult to contain in irregularly shaped counties. Figure 18 shows the border of Suffolk County, MA, along with its service contour. Due to its high population density, its service contour outlines a five-mile thick buffer around the county.

¹¹ 09/18/02 NPSTC Council Meeting, as well as the National Coordination Committee 09/19/02 Implementation Subcommittee and 09/20/02 General Session Meetings.

¹² APPENDIX O, Simplified 700 MHz Pre-assignment Rules Recommendation

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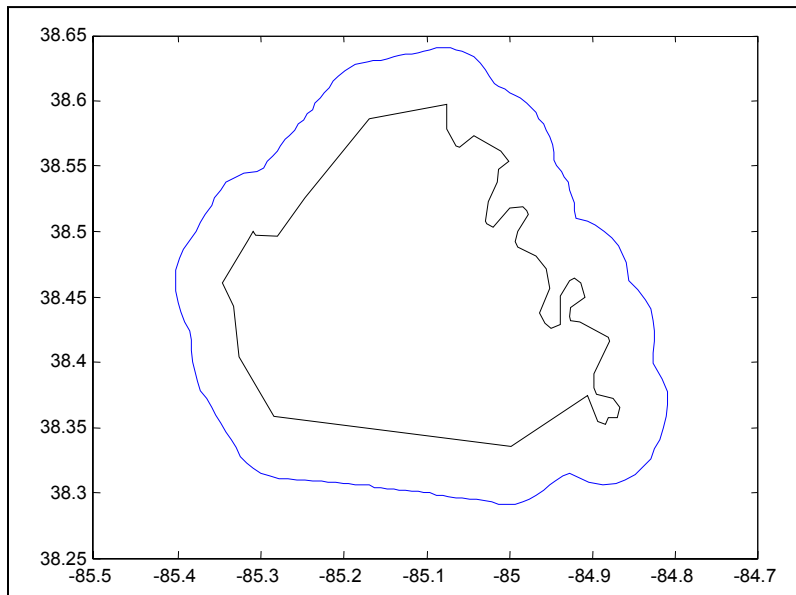


Figure 17, County Border (black) and Service Contour (blue)

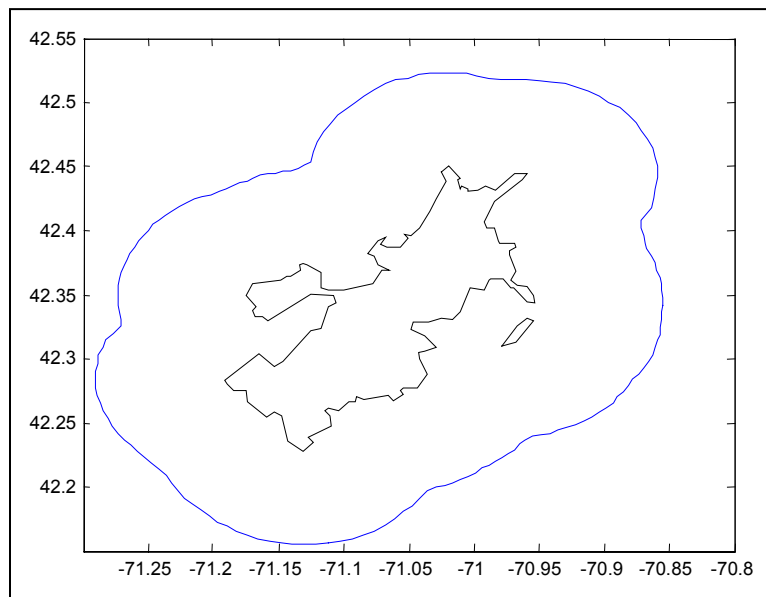


Figure 18, Suffolk County Border (black) and Service Contour (blue)

5.2 Interference Contours

The interference contour for each county represents the geographic region in which there is likely to be measurable interference received from the public-safety radio system of the county. These contours incorporate both the shape of the county and its terrain characteristics.

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The signal from a transmitter is only expected to provide reliable communications within the borders of its home county, but there is little chance that the energy produced will completely drop to zero precisely at the county border. This spillover energy can interfere with co-channel and adjacent signals in neighboring counties. In order to determine which counties would experience co-channel interference from a given county, an interference contour was created (for each county) that identifies its area of interference.

The first step to creating an interference contour is to establish a 50-km buffer around the county border in order to allow the field strength of a transmitter placed anywhere in the county to diminish¹³. A buffer such as this is shown in Figure 19. Under ideal circumstances¹⁴, this buffer could itself act as the interference contour, but this alone does not take into account the topography of the county, only its shape. In the less ideal, more realistic situation in which transmitters are placed at the highest points in the county, the interference can extend well beyond this 50-km buffer. This situation is taken into account by the terrain utilization methods that will be described in Section 5.2.1.

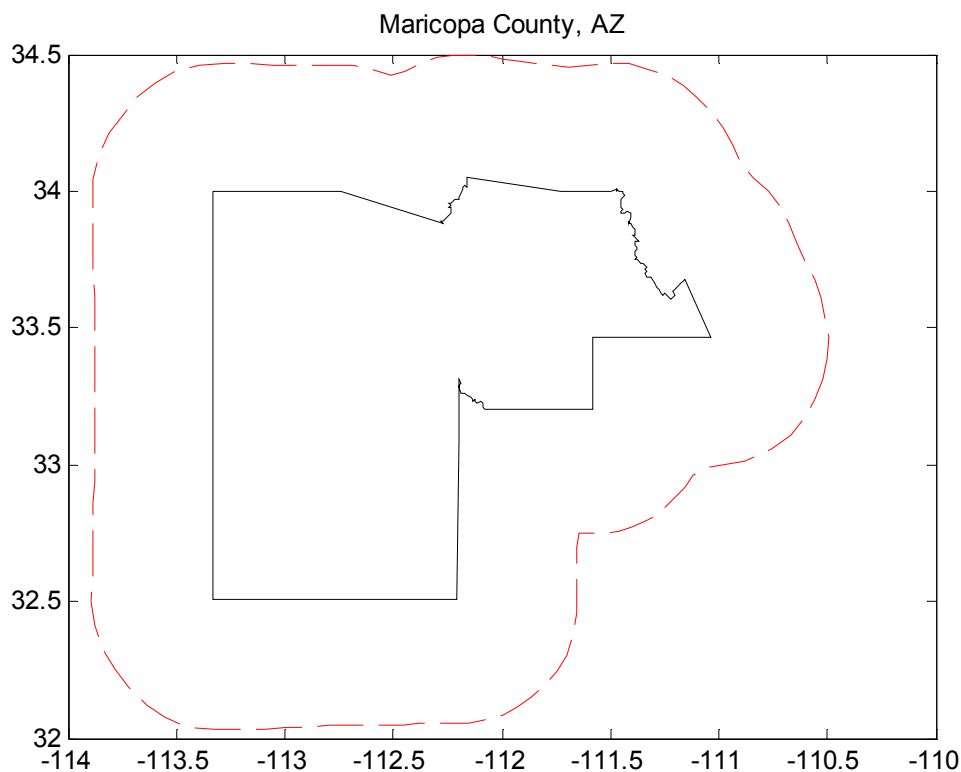


Figure 19, Maricopa County AZ and a 50-km Buffer around It

¹³ On the order of 125 dB of free space path loss alone.

¹⁴ i.e. with significant terrain isolation

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5.2.1 Terrain Utilization

Modeling the interference from a given transmitter site is straightforward. However, trying to generalize the interference potential of a county — without knowledge of the actual transmitter sites that will be used, is much more difficult. Since the transmitters may be located anywhere within the county, the effects of placing the transmitters at the highest points within the county were studied. This is a realistic approach, because transmitters are traditionally placed on top of hills to allow their signals to propagate as far as possible with minimal blockage. For the same reason, placing the transmitters on the highest peaks represents a worst-case scenario when studying an area of interference: signals may propagate far outside the desired area when there are no higher hills to block the transmission.

For each county, topographic data with a grid spacing of 30 arc seconds was used to find the highest point within the county. Then the second highest point was found, excluding the area within a 125-km radius of the highest point. That area was then also eliminated from consideration. The process of finding the highest locations continues in this manner until the highest points are found for the whole county, with these points distributed fairly uniformly throughout the county area. The following provides an example of contour generation for Maricopa County, Arizona; the methodology is the same for the remaining counties.

The steps to find the transmitter locations are illustrated in Figure 20, Figure 21, and Figure 22. In Figure 20, the highest point (red) in Maricopa County AZ is found. Once the circled area is eliminated from consideration, the pink point becomes the next highest in the county. In Figure 21, a 125-km exclusionary zone is created around the second point, and the next highest point is then found. At this point, three representative high points have been selected, and the entire county is included within the regions surrounding those points (see Figure 22).

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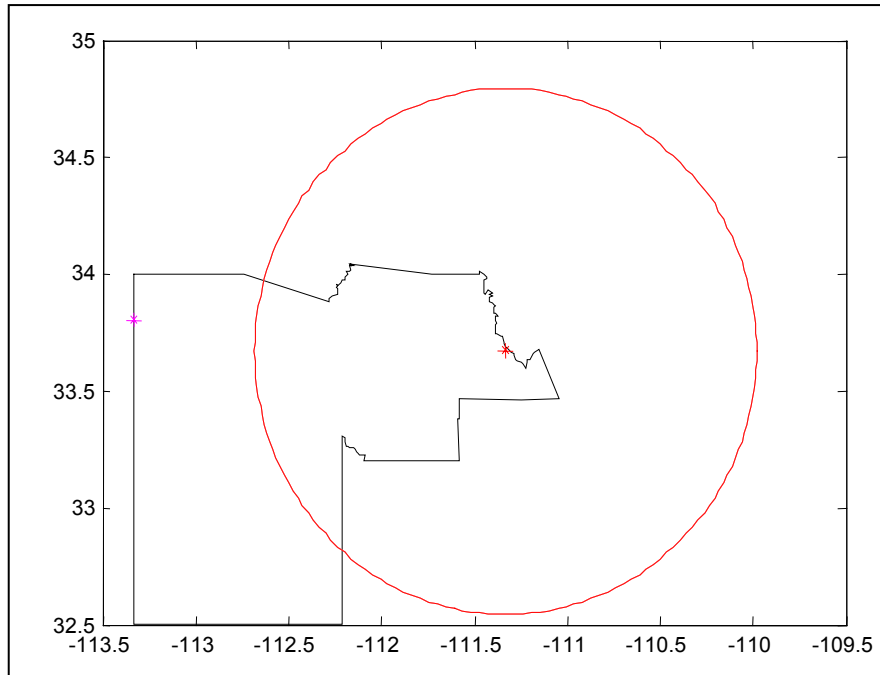


Figure 20, The Highest Point (Red) in Maricopa County AZ

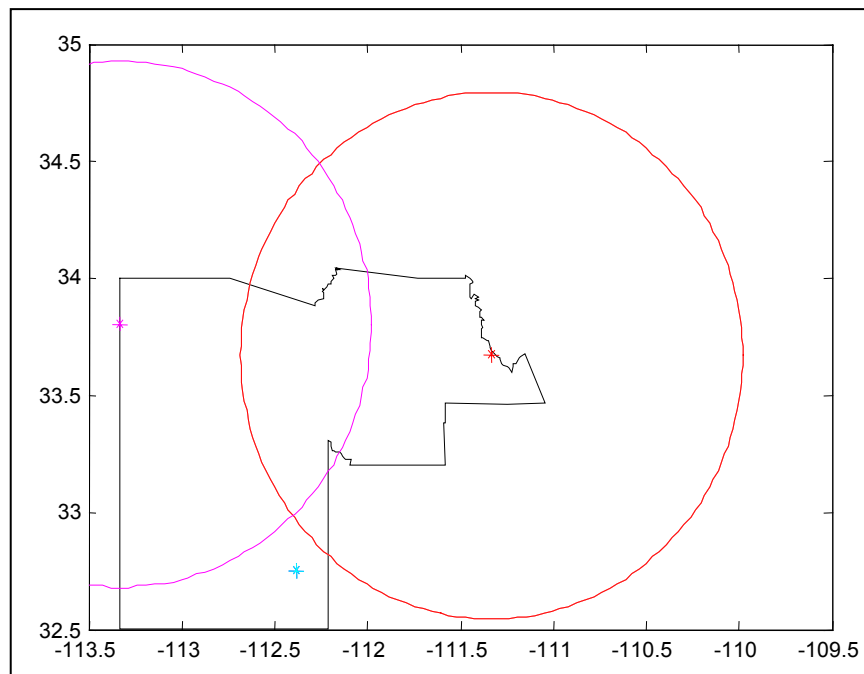


Figure 21, The Next Highest Point in Maricopa County AZ (Separated by 125 km)

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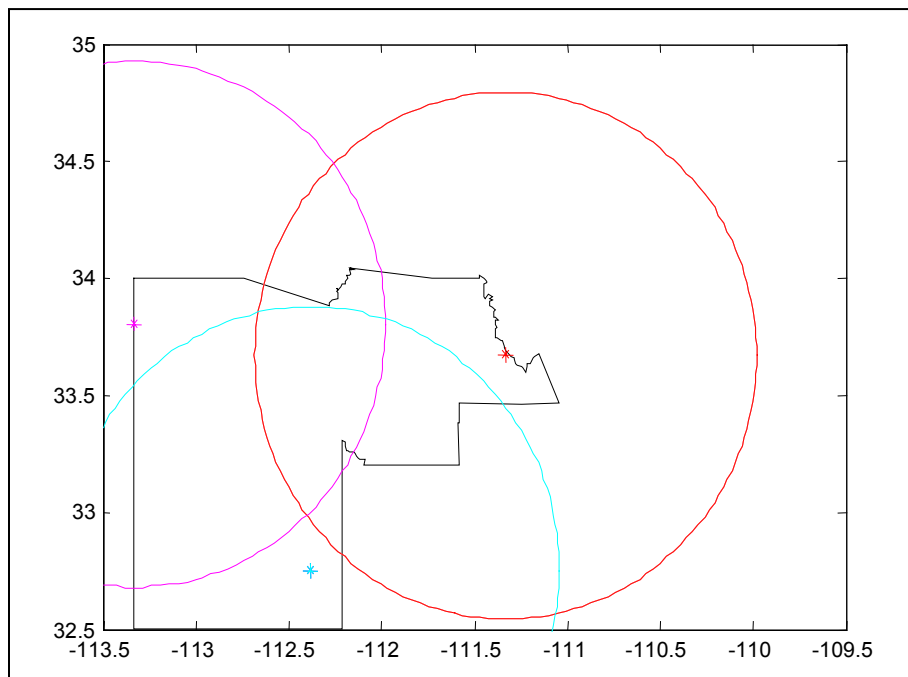


Figure 22, The Three Highest Points in Maricopa County AZ (Separated by 125 km)

Once the representative transmitter locations have been found for the county, propagation modeling commences. An algorithm based¹⁵ on the Okumura-Hata-Davidson propagation model analyzed the field strengths over the terrain from hypothetical transmitters placed at these high points, using the same topographic data previously mentioned. These hypothetical transmitters were set to 30 meters above ground level (AGL), with 785-MHz operation and an ERP of 54-dBm ERP (250W). The mobile receivers were assumed to be 1.5 meters AGL. The field strength was calculated at regular intervals along rays radiating from the transmitter; and portions of the ray that were above a threshold of -128 dBm (~5 dBu into a half wave dipole) were identified. An empirical signal-power distribution was created, and the interference range along any transmitter’s radial was set to the distance at which the 99% of the signal values were less than -128 dBm. This was done for 120 radials per transmitter, imparting each transmitter’s interference contour with a 3-degree angular resolution.

Figure 23 shows the field strengths along one radial radiating from Maricopa County’s highest point. The pink dots highlight the portions above the threshold. The green circle is at the 99th percentile of the pink dots. The interference contour will go through this green circle, indicating that the area to the left of the green circle should expect interference, even though there is a significant area starting at 76 km out in which the field strength is below the threshold. If the contour point were placed at 76 km when the field strength first dips below the threshold, there would be a large patch of interference outside the interference contour. As it is, the interference contour will encompass almost all of the interference. The area to the right of the green circle

¹⁵ This model uses the TSB-88B version of the OHD open model, with additional shadowing losses incorporated that use terrain and primary obstacle, knife-edge diffraction losses.

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will be outside the contour, even though there is one tiny spot in which the field strength is just above the threshold. However, any such spot would be very insignificant in area.

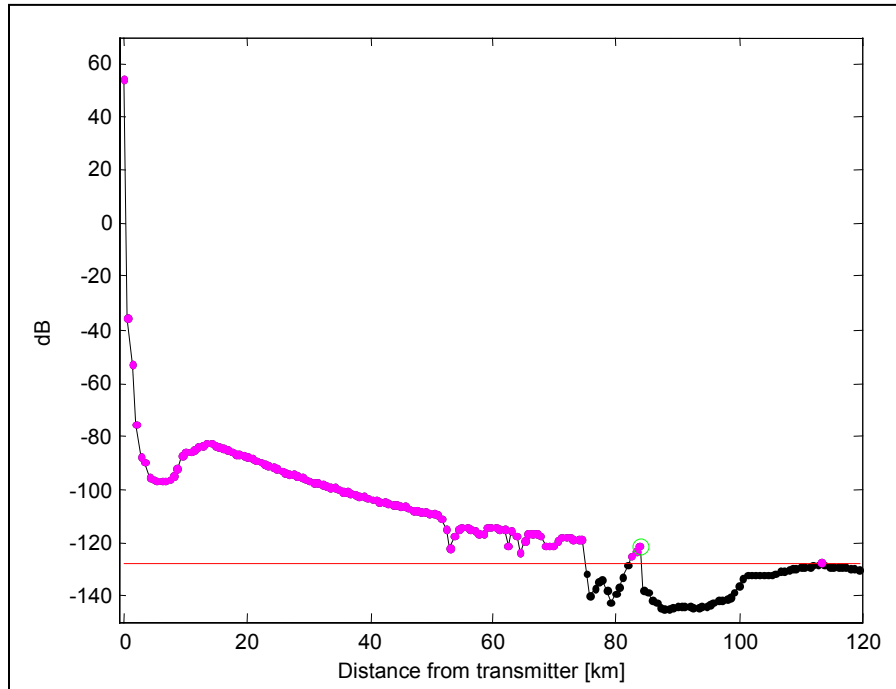


Figure 23, Power along One Radial from the Highest Point

The inference distance points for each transmitter are then connected to form a closed polygon, as shown in Figure 24. The pink dots are spots at which the simulated interference power is greater than -128 dBm. The green dotted contour outlines the polygon, and the area covered by the pink dots.

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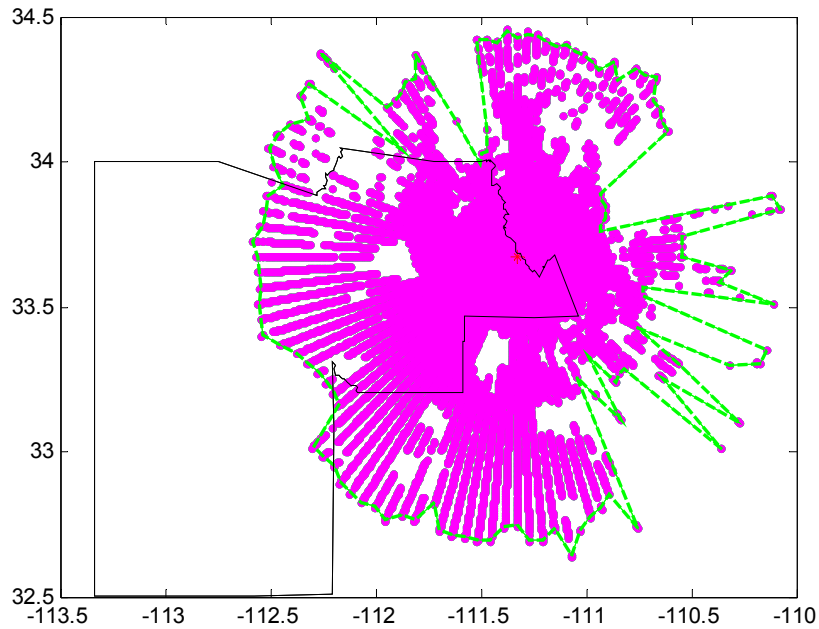


Figure 24, Closing the Interference Polygon for this Transmitter

This is then repeated for each of the County's other transmitter locations (see Figure 25), and the 50 km buffer (see subsection 5.2) is added to account for the possibility that a transmitter could be located away from the highest points and near the county border (Figure 26).

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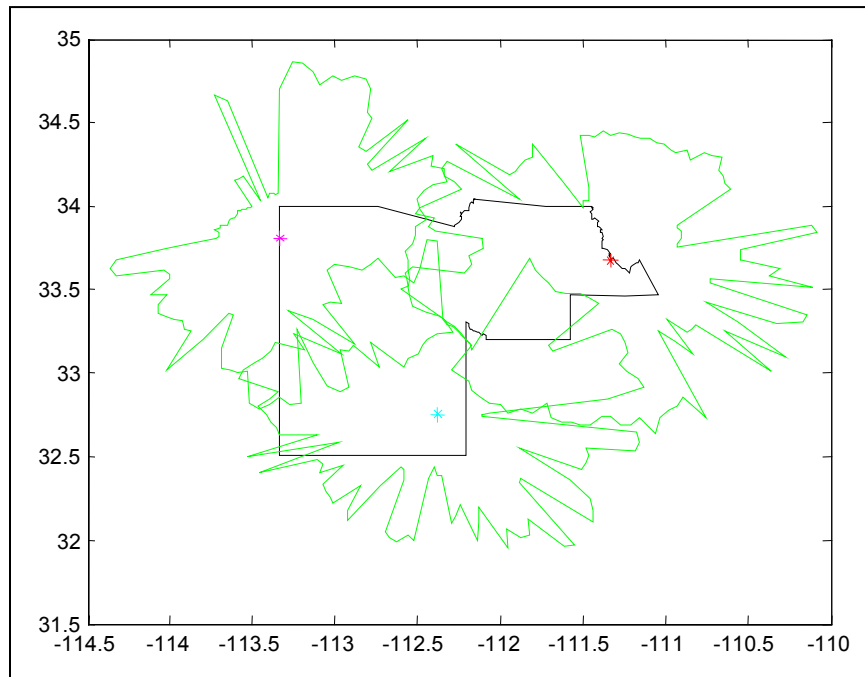


Figure 25, Interference Counters for the Three Representative Transmitters

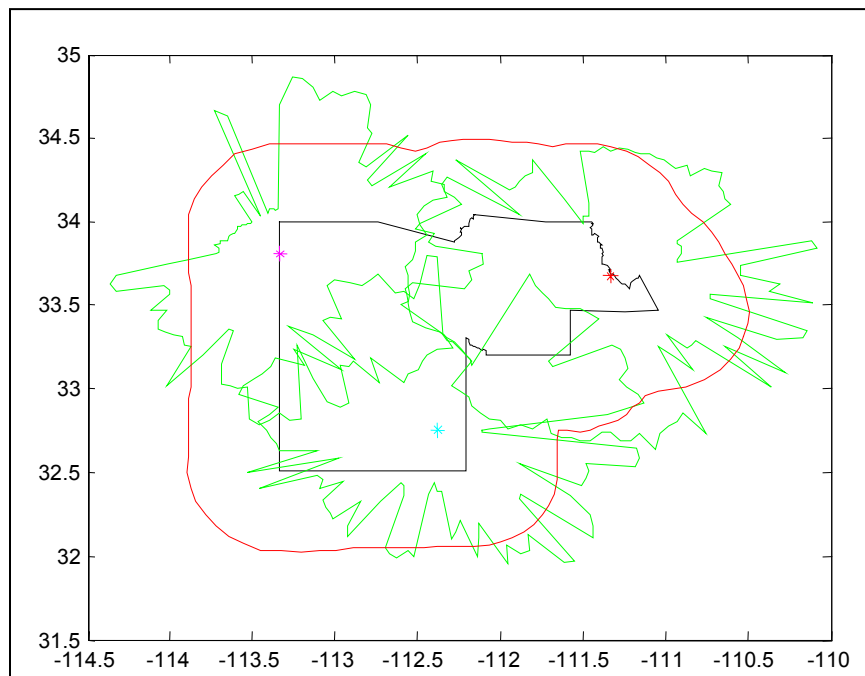


Figure 26, Interference Counters for the Representative Transmitters, and the 50-km Buffer

Finally, the County's final interference contour becomes the union of the polygons of each transmitter's interference polygon and the 50-km buffer zone (Figure 27). Thus, the final contour

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represents a good model for a worst-case scenario. The final Service and Interference Contours for Maricopa County are provided in Figure 28, along with a terrain underlay.

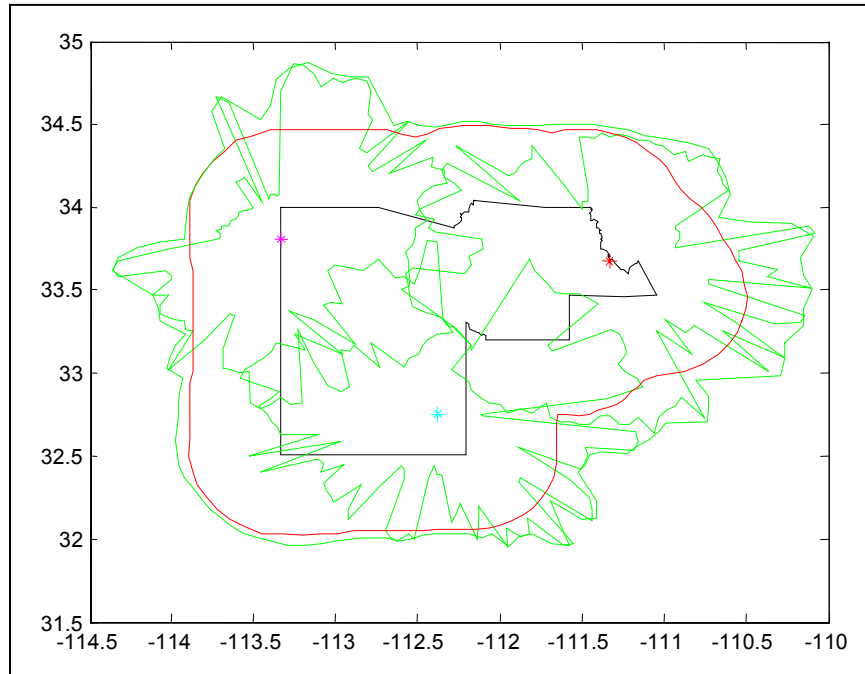


Figure 27, Final Contour (Solid Green) with its Constituent Parts (Dotted Green and Red)

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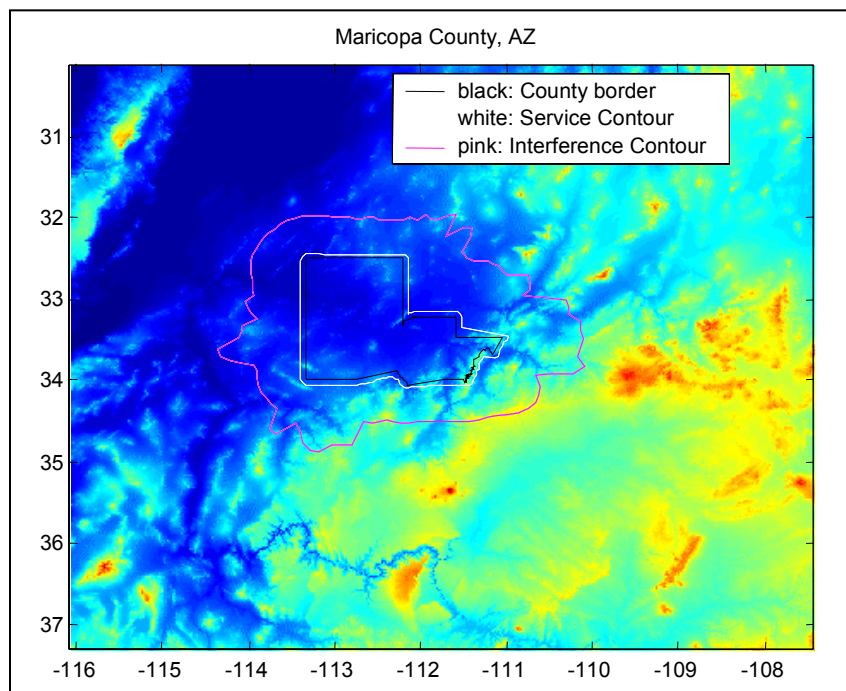


Figure 28, Maricopa County, with Service (White) and Interference (Red) Contours and Terrain

Additional illustrations of the contouring methodology are shown (in Figure 29, Figure 30, and Figure 31) for San Francisco County and (in Figure 32, Figure 33, and Figure 34) for Orange County (both counties in CA). These were selected to illustrate the effects of both terrain blocking and over-water propagation. Figure 29 and Figure 32 show the propagation results for each area using the representative transmitters. Figure 30 and Figure 33 show the interference locations that were identified from the propagation models, and the formation of the preliminary interference contours. Figure 31 and Figure 34 show how the final contours results from the union of the initial interference contours (generated through the propagation model) and the 50-km buffer. It is clear that this contouring does exactly what it was intended to do — capture the interference potential of the county by examining the local terrain and exploiting terrain blocking to increase channel reuse.

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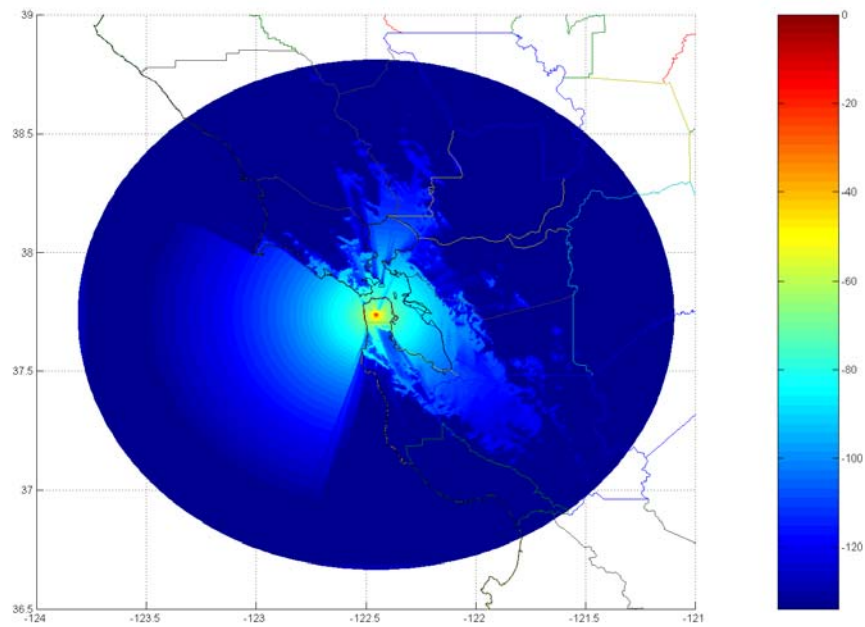


Figure 29, San Diego County (CA), Propagation Model Levels (dBm)

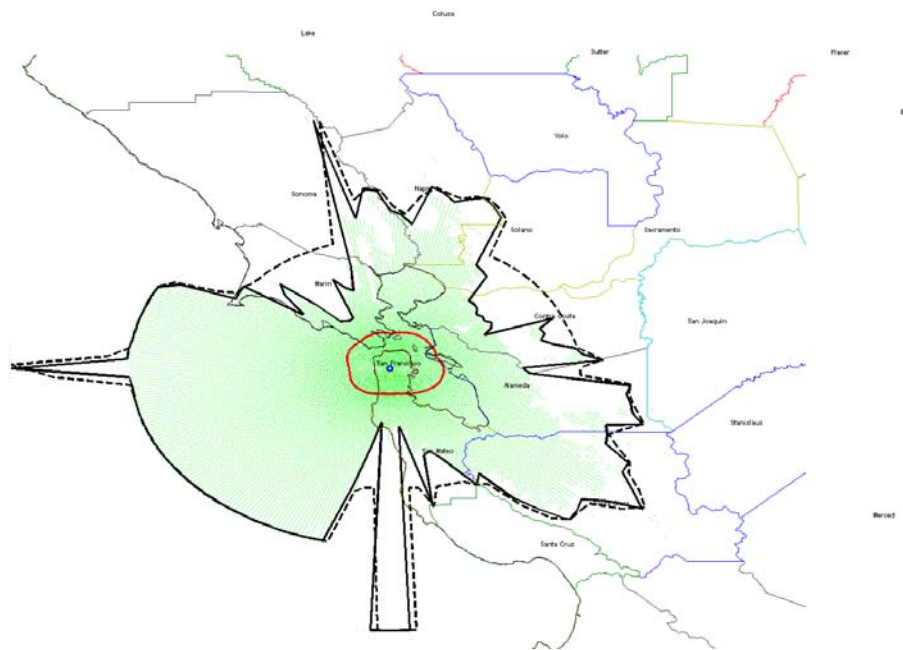


Figure 30, San Diego County (CA), Interference Locations and Contours

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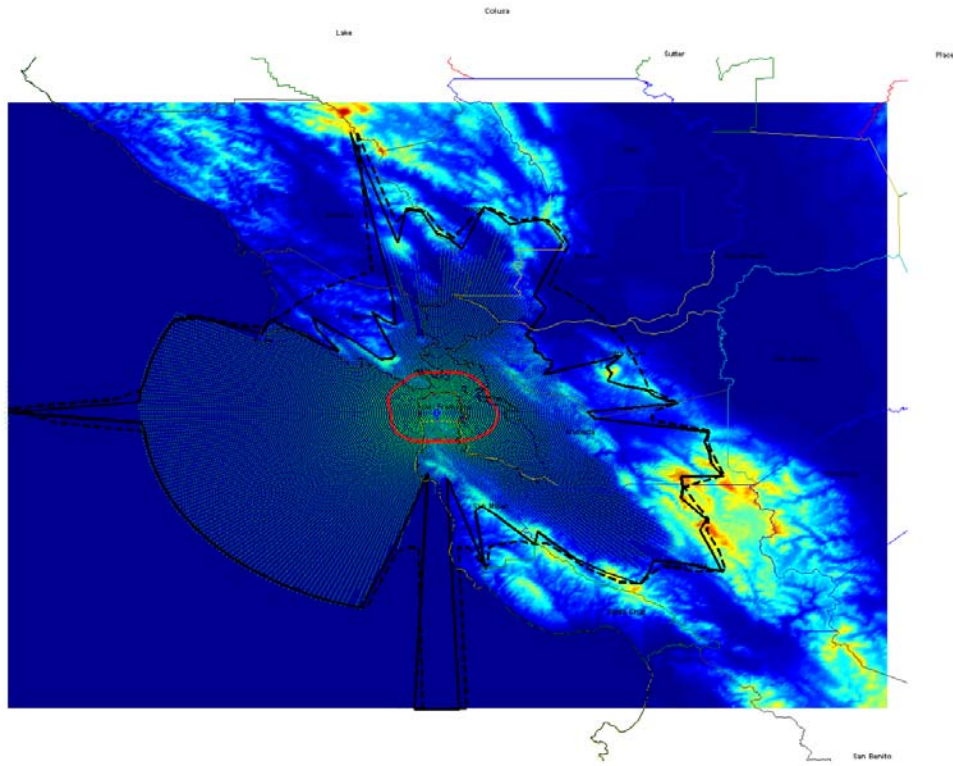


Figure 31, San Diego County (CA), Interference Contours w/Terrain Underlay

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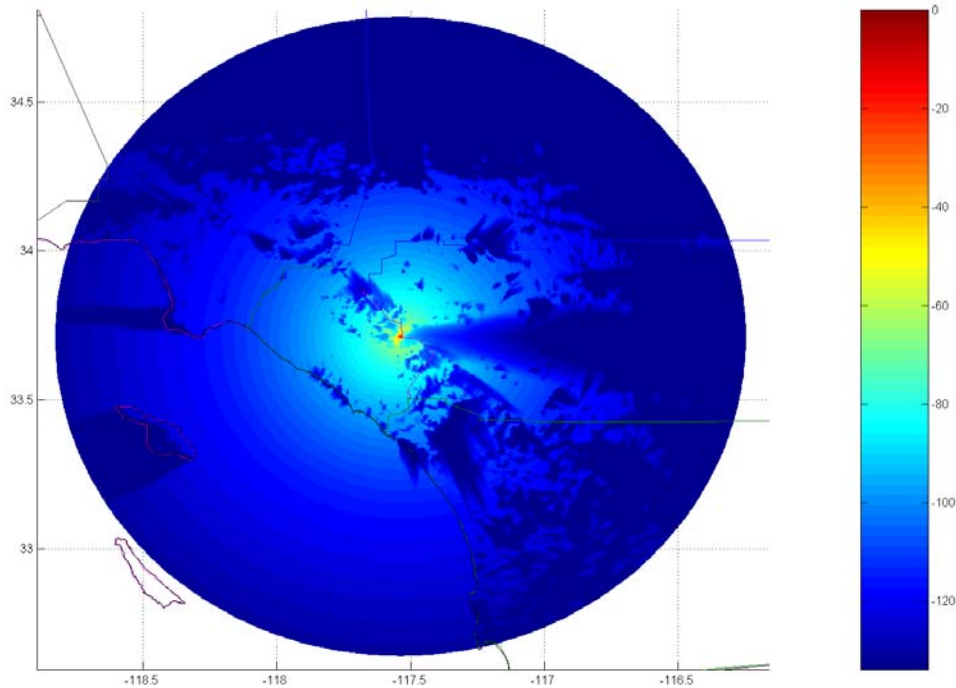


Figure 32, Orange County (CA), Propagation Model Levels (dBm)

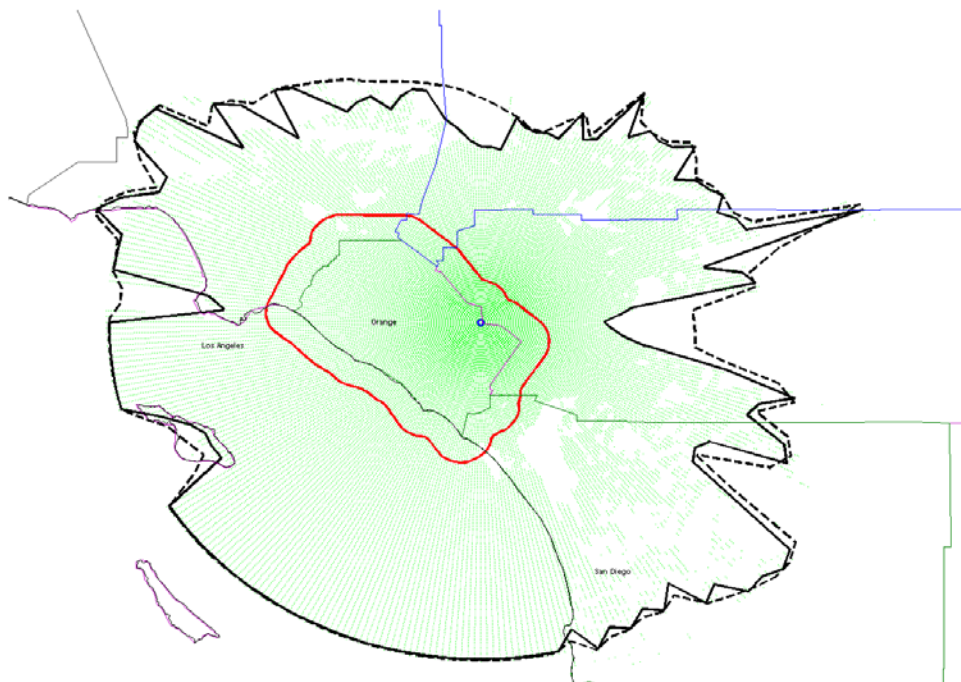


Figure 33, Orange County (CA), Interference Locations and Contours

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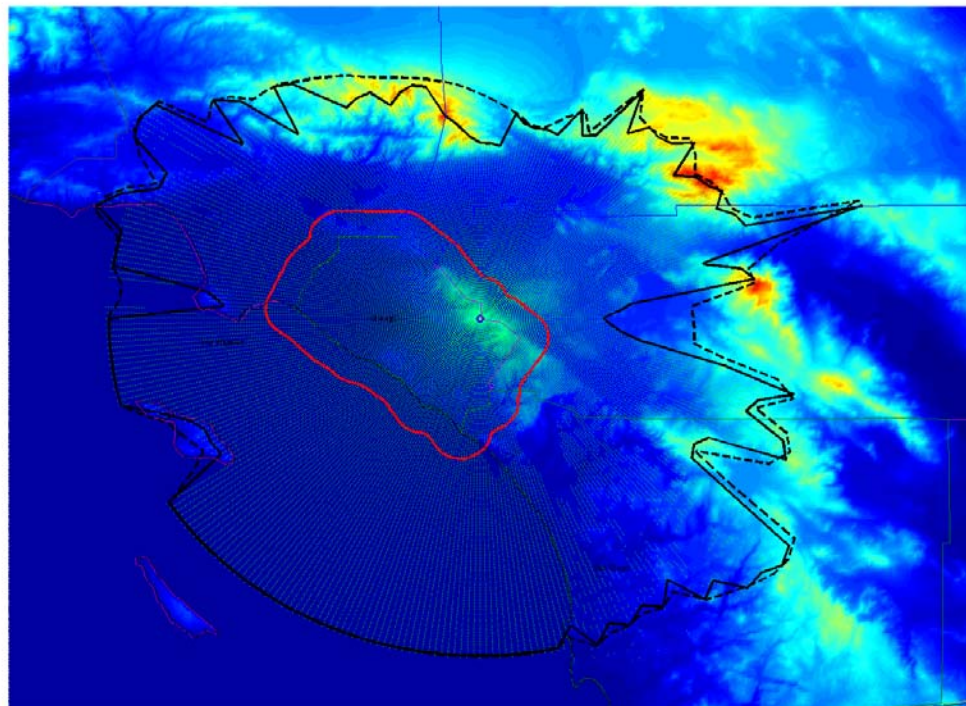


Figure 34, Orange County (CA), Interference Contours w/Terrain Underlay

5.2.2 Contour Parameters and Characteristics

The previous subsection examined how terrain was used in the contouring methodology in order to define the interference contour for each county. The method used was a new contouring technique that utilized ray tracing and radial propagation modeling to define interference and to identify and exploit terrain blocking and shadowing effects. This subsection briefly examines the resulting reliability that could be expected using the parameters chosen to generate these contours.

The threshold level chosen for the interference contours was -128 dBm, or about 5 dBu with a half-wave dipole. It is the comparison of these interference contours to the set of service contours¹⁶ (and vice-versa) that define whether a co-channel pool assignment is allowable. The use of 40 dBu for service contours and 5 dBu for co-channel interference contours are consistent with standard 800-MHz frequency coordination, as well as with recommendations for 700-MHz operation¹⁷. However, the 40 dBu and 5 dBu are normally used as median contour levels. Recall from subsection 5.2.1 that the interference contour level is set at -128 dBm at the 99th percentile.

¹⁶ See 5.1

¹⁷ APPENDIX O, Simplified 700 MHz Pre-assignment Rules Recommendation

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Furthermore the concept of an empirical distribution of power levels along a radial is not equivalent to median values at the contour.

In order to ascertain the noise and interference reliability of these values, one needs to determine an approximate equivalent value of contour reliability corresponding to the 99th percentile threshold at 5 dBu. Since the distribution of power values is evenly distributed along each radial, the 99th percentile clearly indicates more a measure of “area coverage” than “contour coverage.” Assuming that the radial propagation model is unbiased, and errors are uncorrelated in distance/area, the contour coverage associated with 99% area coverage is approximately¹⁸ 97% for typical lognormal variance levels. This corresponds to a single-sided normal Z value of -1.85σ . For a desired edge-of-contour (EOC) service level of 40 dBu, Table 2 and Figure 35 illustrate the expected noise, interference, and noise-plus-interference reliability levels, assuming a receiver noise floor of -126 dBm¹⁹ and a faded-channel performance criterion (CPC) of 18 dB²⁰. It is clear that the intent here is to keep the interference insignificant with respect to noise levels, and to keep the joint probability of noise and interference as high as possible. This objective was clearly met though the selection of these parameters.

Table 2, Expected Noise, Interference, and Aggregate Reliability for Pool Assignments

Base Lognormal Stdev (dB)	Median Interference Level at EOC (dBm)	Noise Reliability* (%)	S/I Level* (dB)	Interference Reliability Margin (dB)	Interference Reliability (Nint = 1) (%)	Interference Probability (Nint = 1) (%)	Aggregate Reliability (N+1) (%)
5	-137.4	99.9	44.4	26.4	100.0	0.0	99.9
6	-139.3	99.4	46.3	28.3	100.0	0.0	99.4
7	-141.1	98.5	48.1	30.1	99.9	0.1	98.4
8	-143.0	97.1	50.0	32.0	99.8	0.2	97.1
9	-144.9	95.5	51.9	33.9	99.6	0.4	95.4
10	-146.8	93.6	53.8	35.8	99.4	0.6	93.6
11	-148.6	91.7	55.6	37.6	99.2	0.8	91.6
12	-150.5	89.8	57.5	39.5	98.6	1.4	89.7

*Desired EOC Level of 40 dBu, Receiver Noise Floor of -126.2 dBm; (ex. C4FM ENBW of 6 kHz, NF = 10 dB)

¹⁸ For more detail, see:

“Foundations of Mobile Radio Engineering,” Michael D. Yacoub, CRC Press 1993, Section 3.5.1;

Telecommunications Industry Association, Technical Service Bulletin TSB-88A, “WIRELESS COMMUNICATIONS SYSTEMS, PERFORMANCE IN NOISE- AND INTERFERENCE-LIMITED SITUATIONS, RECOMMENDED METHODS FOR TECHNOLOGY-INDEPENDENT MODELING, SIMULATION, AND VERIFICATION,” Section 4.4.2; and

“Land Mobile Radio Systems Engineering,” Gary C. Hess, Artech House 1993, Section 10.

¹⁹ This is typical of a Project25 (P25) Phase I, C4FM Receiver, with an ENBW of 6 kHz and a 10-dB overall noise figure.

²⁰ Corresponding to DAQ 3.4 for a P25 Phase I receiver.

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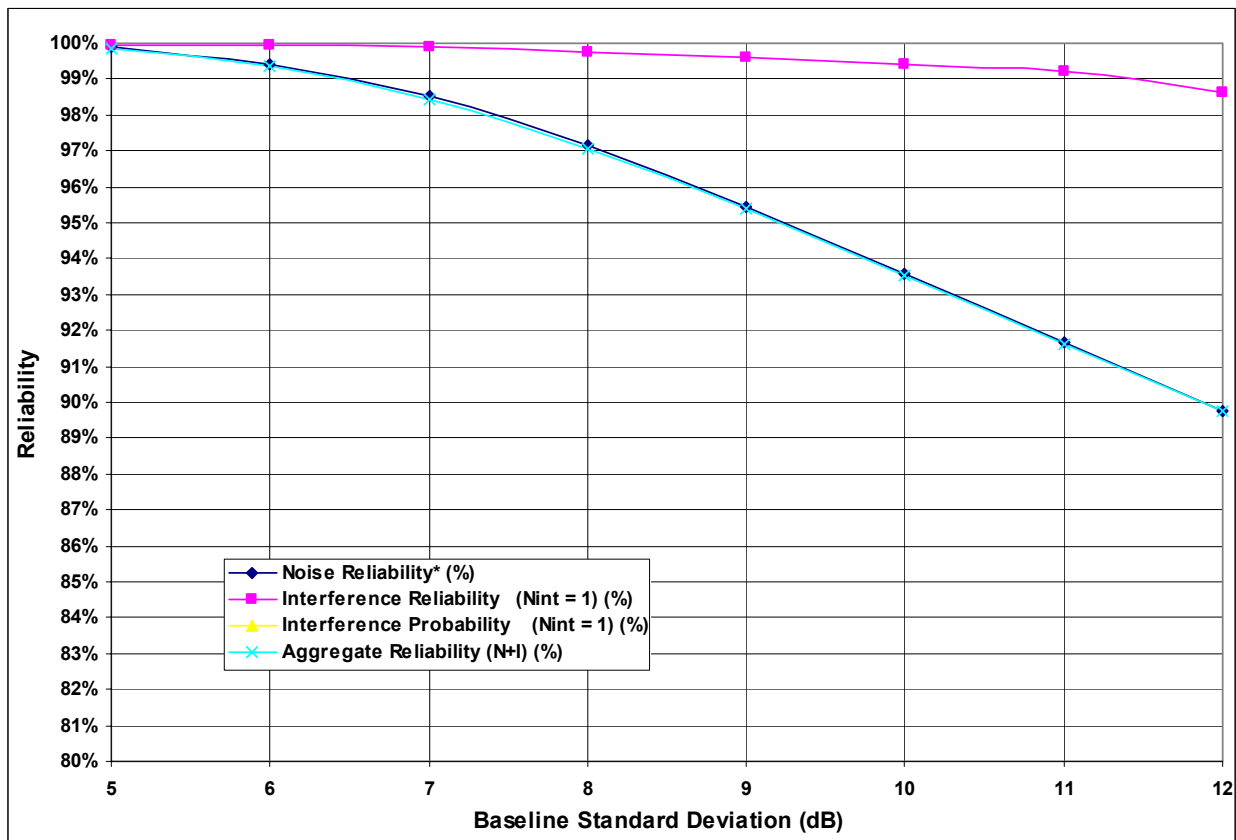


Figure 35, Expected Noise, Interference, and Aggregate Reliability for Pool Assignments

5.3 Examples of Final Results

This subsection presents examples of final contours. In these examples, a county’s border is depicted in black, its service contour in white, and its interference contour in pink. The background color depicts terrain elevation in meters according to the color bar legend on each picture.

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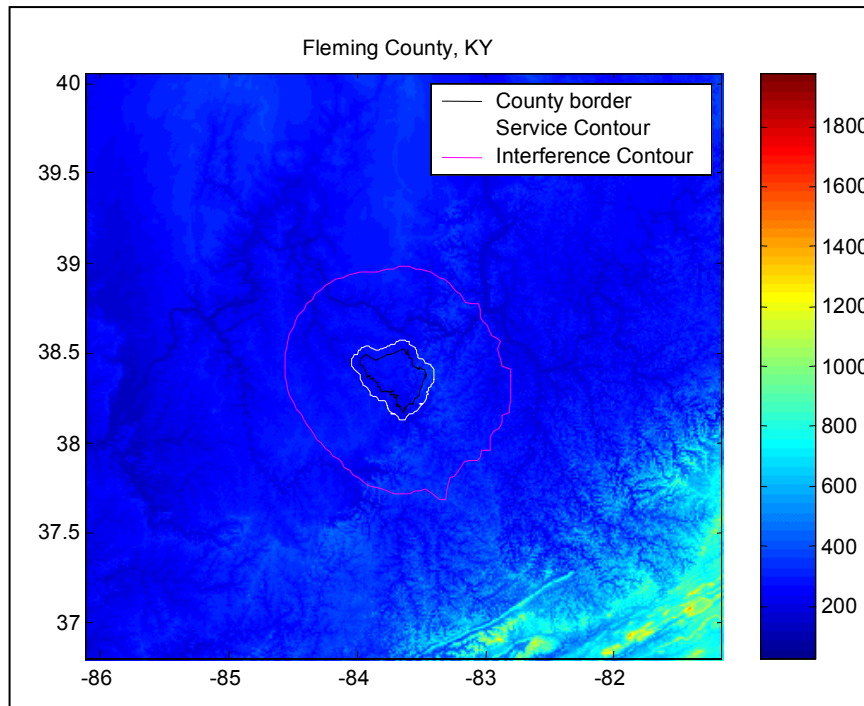


Figure 36, Final Contours, Fleming County KY

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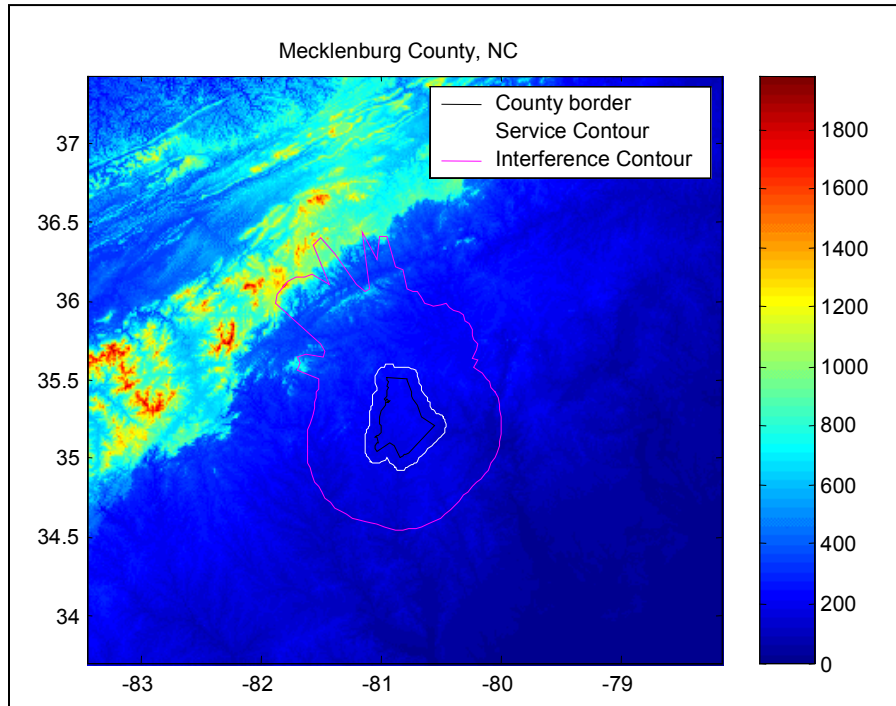


Figure 37, Final Contours, Mecklenburg County NC

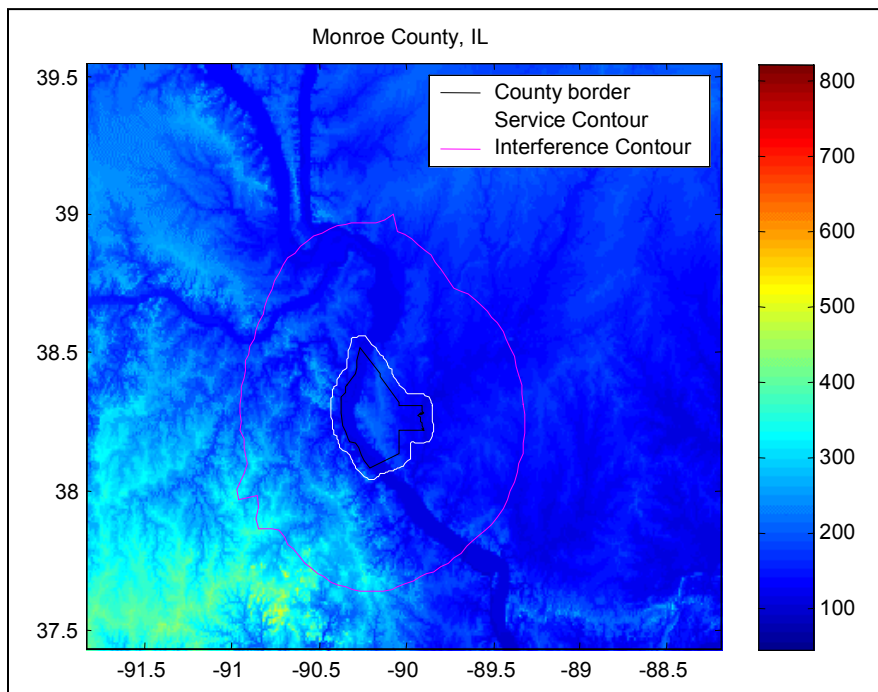


Figure 38, Final Contours, Monroe County IL

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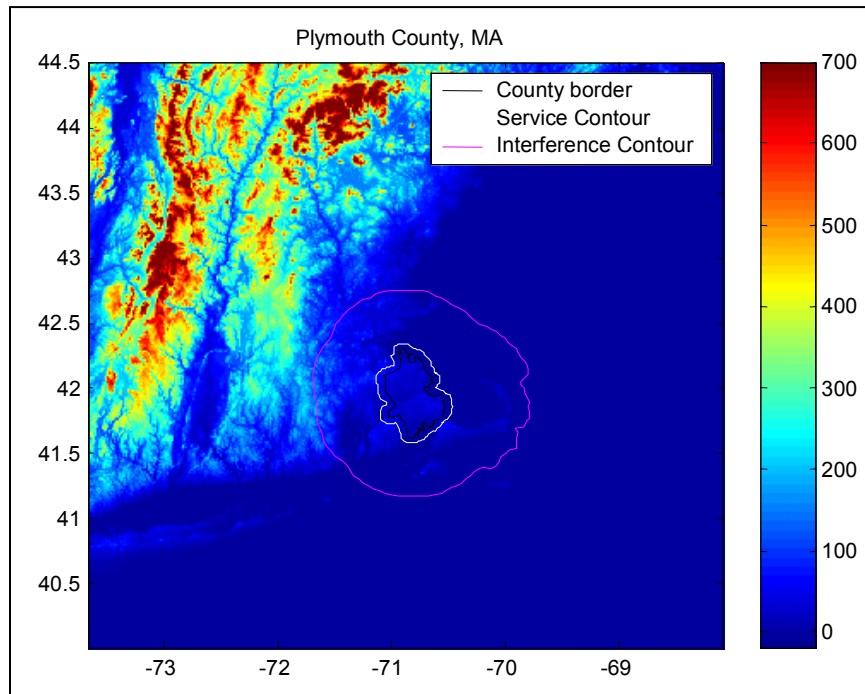


Figure 39, Final Contours, Plymouth County MA

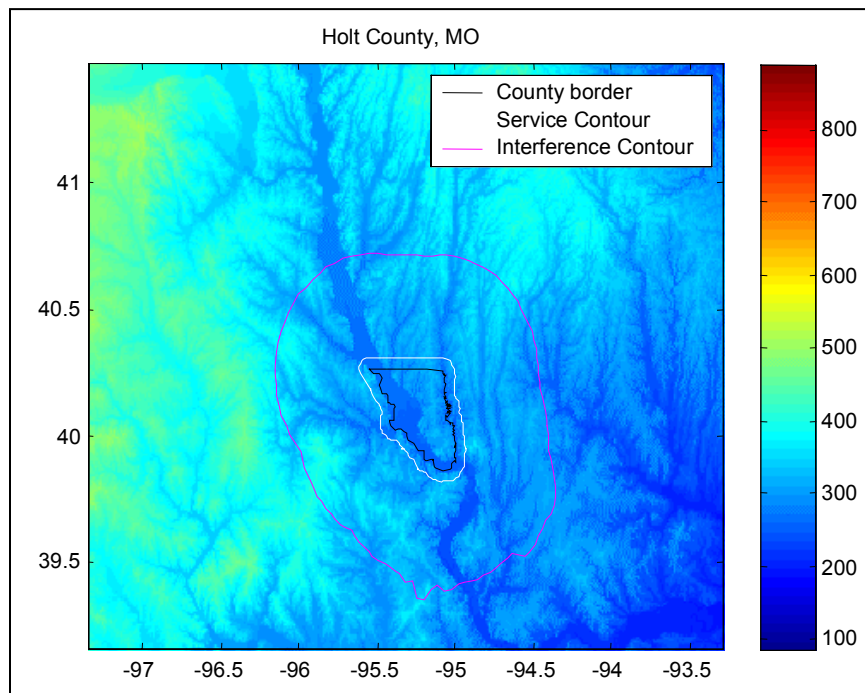


Figure 40, Final Contours, Holt County MO

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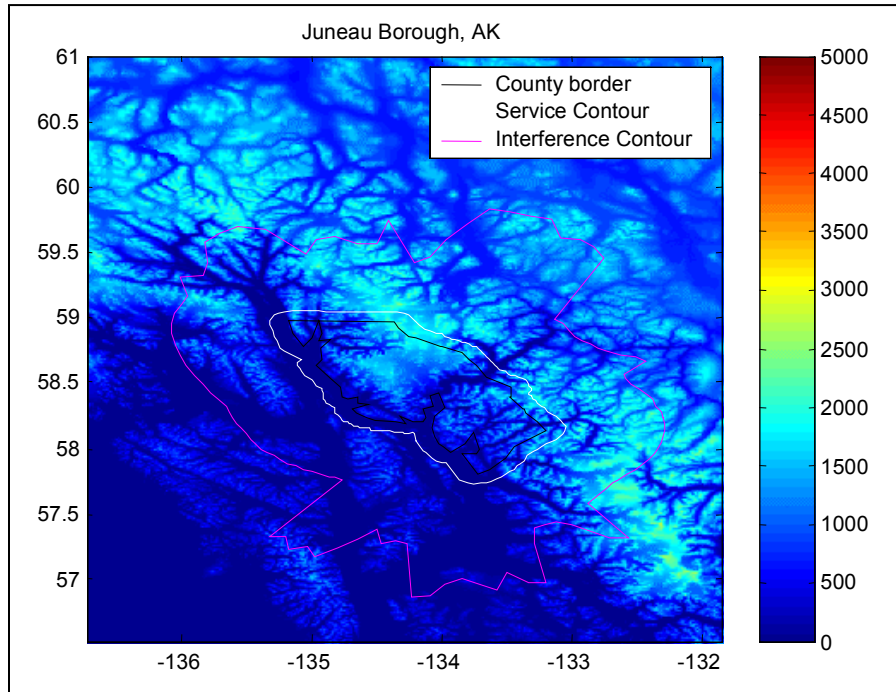


Figure 41, Final Contours, Juneau Borough AK

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6. GENERATION AND OPTIMIZATION OF CHANNEL ASSIGNMENTS

The methodology for making the channel selections for the pool allotments is documented in this section. First, an overview of the frequency assignment problem is given. Then, an introduction to the type and degree of constraints is provided. The remainder of this section focuses on the formulation of the specific problem and on the methodology applied to generate the 700-MHz pool allotments.

6.1 Overview of the Assignment Problem

Given a set of sites $S = \{s_1, s_2, \dots, s_N\}$ and a set of frequencies $F = \{f_1, f_2, \dots, f_K\}$, a possible assignment \underline{A} is an $N \times K$ binary matrix. $A_{ij} = 1$ means that site s_i is assigned frequency f_j , while $A_{ij} = 0$ means site s_i is not assigned frequency f_j . The indices $i = (1 \dots K)$ are referred to as *channels*.

The number of possible assignments is 2^{NK} . An obvious observation is that, even for modest sizes of N and K , a brute force exhaustive search over all possible assignments is intractable; there is just not enough computing power and time available given today's state of technology. For the problem considered here, $N = 3,223$ and $K = 154$.

Not all-possible assignments are *permissible*. There are a priori constraints on possible assignments that must be observed in order for them to be permissible. These constraints are described in detail in subsection 6.2, which follows. For each permissible assignment, there will also be a way to compute a figure of merit. This is discussed in subsection 6.3.

The Assignment Problem can be defined as the problem of finding a permissible assignment with the highest possible figure of merit. The number of permissible assignments, like the number of possible assignments, is again much too large to permit an exhaustive search. An absolute optimum solution to the Frequency Assignment Problem (FAP) cannot be guaranteed by any known method.

6.2 Assignment Constraints

This subsection describes the constraints that a possible assignment must satisfy in order to be a permissible assignment. It starts with a preliminary definition of the frequency vector $F = \{f_1, f_2, \dots, f_K\}$. Without loss of generality, one can assume that the frequencies are sorted from smallest to greatest and can say that consecutive frequencies are *adjacent* if they differ by at most some fixed value. For the problem considered here, frequencies that differ by 25 kHz are considered adjacent. The figure below shows the frequency vector F , which corresponds to the mapping of the General Use 700-MHz Public Safety channel set.

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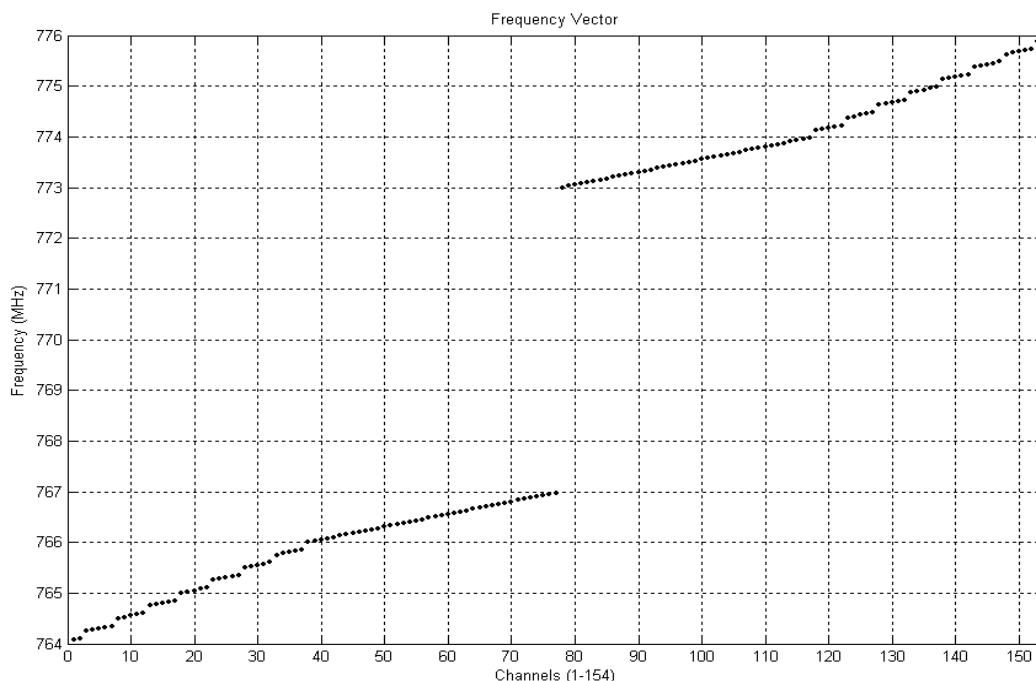


Figure 42, Block to Frequency Mapping for the Assignments

6.2.1 Single-Site Constraint

Two frequencies assigned to the same site must have a minimum separation. In general, the minimum separation can vary from site to site. For the problem considered here, the minimum separation is 250 kHz for each site. Thus, any two frequencies assigned to the same site cannot differ by less than 250 kHz. An analysis of this constraint, given the available frequency vector, reveals that the maximum number of frequencies/blocks that any site can be assigned is 24 out of the possible 154.

6.2.2 Site-Pair Constraints

If two different sites are assigned the same frequency or an adjacent frequency, unacceptable interference may result. It is interesting to note that this interference is not necessarily symmetric. For example, the first site may cause interference to the second site, but at the same time experience no interference from the second site. However, site-pair constraints must be symmetric in order to eliminate all interference; two sites can only either share, or not share, co-channel blocks. Therefore, two sites can only either share, or not share, adjacent-channel blocks.

In order to characterize all site pair constraints, two binary and symmetric *permission matrices* CO and ADJ are defined. CO is the $N \times N$ co-channel permission matrix; and, for any sites i and j , CO_{ij} is 1 if and only if sites i and j may be assigned the same frequency. Similarly, ADJ is the

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adjacent-channel permission matrix, which has values of one where the site pairs may be assigned adjacent frequencies.

The CO and ADJ matrices are derived from the Service and Interference Contours described in section 5. If the service area from site i intersects with the interference area of site j , or if the service area of site j intersects the interference area of site i , then $CO_{i,j} = 0$. If the service area from site i intersects with the service area of site j , then $ADJ_{i,j} = 0$. The diagonal of the CO matrix is unity (1), while the diagonal of the ADJ matrix is all-zero, i.e. $diag(CO) = \underline{1}_N$ and $diag(ADJ) = \underline{0}_N$.

Since service area is a subset of interference area, it follows that $CO_{i,j}$ is always less than or equal to $ADJ_{i,j}$ except on the diagonal ($i \neq j$). In other words, whenever two different sites may be assigned the same frequency, they may also be assigned adjacent frequencies but not vice versa.

6.3 Figure of Merit

The binary $N \times K$ matrix \underline{A} is a permissible assignment if it satisfies all the predefined constraints, i.e. the single-site constraints and the site-pair constraints (which are comprised of the co-channel constraints and the adjacent-channel constraints). A trivial example of a permissible assignment is the matrix of all zeros, $\underline{0}_{N,N}$. Clearly this assignment satisfies all constraints; but, of course, it is useless because no frequencies/blocks are assigned.

What is needed is a criterion that can be used to compare two different permissible assignments \underline{A}_1 and \underline{A}_2 to determine which is better. A simple criterion might be the total number of assignments made. However, using this criterion might leave some sites with no frequencies assigned. Another possibility is to consider the minimum number of frequencies assigned to a given site. Let a_1 and a_2 be formed by summing \underline{A}_1 and \underline{A}_2 along the rows. Next a_1 and a_2 can be sorted independently from smallest to largest. Let i be the first place where a_1 differs from a_2 , then select \underline{A}_1 or \underline{A}_2 according to whether $a_1(i)$ is greater than $a_2(i)$.

A drawback to the above criterion is that it treats all the sites equally, without taking into account that some sites require more frequencies than others. The capacity model described in section 4 addresses the difference in requirements for the sites and needs to be incorporated into the figure-of-merit for the assignments.

The capacity vector, C , is an $N \times 1$ vector of numbers between 0 and 1. Used here is a *normalized capacity*, which is the capacity multiplied by the maximum number of frequencies that a single site could receive without violating the single-site constraint. For the problem considered here, the capacity is multiplied by 24, i.e. $C \in (0, 24]$.

Now for each site one can take the ratio of the number of frequencies given the site to the normalized capacity for that site and call this ratio the *merit* for the given site. Given two assignments \underline{A}_1 and \underline{A}_2 , form the merit vectors m_1 and m_2 and sort them independently. Then choose the assignment according to what happens at the first point at which the sorted merit vectors differ.

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An additional refinement in the merit definition incorporates the desire to achieve a specific minimum desired number of frequencies (nf) for all sites. In the problem under discussion, the number selected was between 4 and 5. This refinement affects the site merit calculation of those sites whose normalized capacity is less than the minimum desired number of frequencies. If a site has a normalized capacity less than nf and if the number of frequencies assigned to that site is also less than nf , then the merit of that site is the number of frequencies assigned to it divided by nf instead of divided by the site's normalized capacity. For example, suppose the normalized capacity is 0.5, $nf = 5$, and the number of frequencies assigned is $[0\ 1\ 2\ 3\ 4\ 5\ 6]$. Then the corresponding merits are $[0.0\ 0.2\ 0.4\ 0.6\ 0.8\ 10.0\ 12.0]$. In this way the merit for sites with small capacity are artificially deflated until they get the minimum desired number of frequencies.

The figure-of-merit comparison of assignments described above forms a total ordering of all possible assignments so that, of all permissible assignments, there is a unique maximum figure of merit, although many different permissible assignments may achieve this maximum. Any assignment that achieves the maximum figure of merit is an absolute optimum solution to the assignment problem. Of course, the problem is such that one cannot know when an absolute optimum solution is achieved and so one must be satisfied with obtaining a practical solution to the problem. The next subsection describes how this was accomplished.

6.4 Generation of Permission Matrices

Before channel blocks could be assigned to counties, it was necessary to determine which counties would experience interference from each of the other counties, if assigned the wrong frequencies. To this end, two symmetric permission matrices were created, in which each row represented one county's ability to share a channel with each other county. The first matrix, CO , represented co-channel sharing permissions, and the second, ADJ , represented adjacent-channel sharing permissions.

6.4.1 Hard Constraint

The matrices used in the final solution to the problem were generated under hard constraints. If the interference contour (see subsection 5.2) of one county overlapped the service contour (see subsection 5.1) of another county at all, then the two counties were denied permission to share a co-channel. For any pair of counties, i and j , the co-channel permission matrix is 1 if and only if the service contour of county i did not intersect the interference contour of county j and the interference contour of county i did not intersect the service contour of county j . Thus the CO matrix is systemic, i.e. $CO(i, j) = CO(j, i)$, and $CO-CO^T = 0_{N,N}$. Despite this, in reality the interference condition may not be symmetric and/or bi-directional. This is illustrated in Figure 43, in which Franklin County's Interference contour (solid green) overlaps Warren County's Service contour (dashed blue). These counties will not be allowed to share a co-channel block, i.e. $CO(Franklin, Warren) = CO(Warren, Franklin) = 0$, even though Warren's Interference contour (dotted green) does not overlap Franklin's service contour (dotted blue).

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Similarly, for a given pair of counties, i and j , the adjacent-channel permission matrix entry is set to 1, allowing adjacent-channel sharing if and only if the service contour of county i does not intersect the service contour of county j .

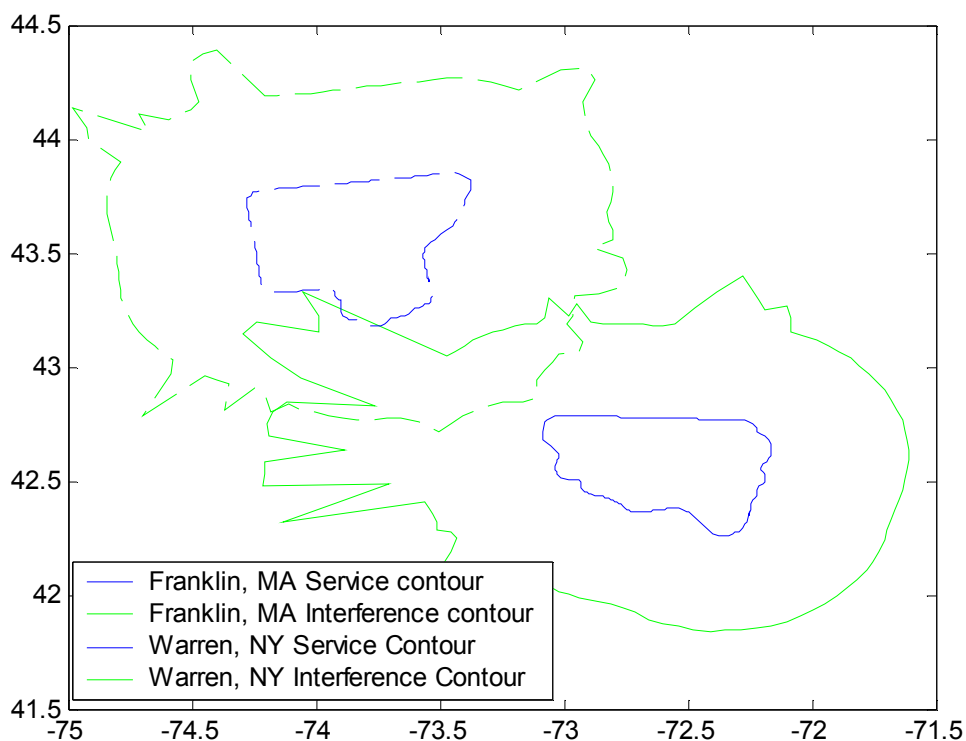


Figure 43, Example of Asymmetric Interference Conditions

These hard constraints play a strong role in generating the allotment pool, since they ultimately limit the availability of spectrum in any given area, regardless of the capacity need. One measure of the total constraint level as a function of location is the sum of the integration along any one dimension of the inverse of the CO and ADJ matrices. A map of these total constraint levels for the continental U.S. is shown in Figure 44.

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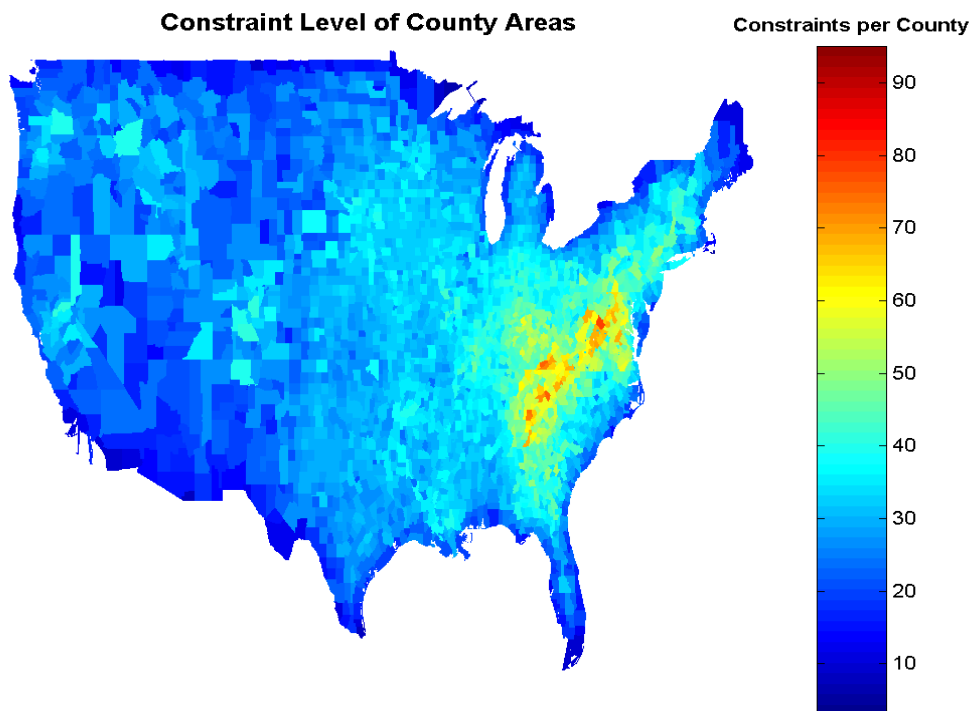


Figure 44, Overall Constraint Levels, Continental U.S.

6.4.2 Soft Constraint Area of Overlap Metric

The goal of the frequency assignment problem is to assign the requested number of frequencies to each county in such a way that no county suffers interference from another county. To that end, co-channel constraints and adjacent-channel constraints were developed. The co-channel constraints demanded that no county be assigned to a certain frequency if its coverage contour intersected with the interference contour of any other county already assigned that frequency, and similarly that no county be assigned to a certain frequency if its interference contour intersected with the coverage contour of any other county already assigned that frequency. In some cases, however, the problem may not be solved under these hard constraints. Soft constraints can be utilized to deal with these difficult cases.

Consider an island made up of small counties and that each requires five channel blocks. Say that some counties receive only three blocks using the best solution generated using the hard constraints. To allot more frequencies, one might take a second look at the contour intersections. If a county were found whose coverage contour just barely overlapped the interference contour of another county, then one might decide to allow the two counties to share a frequency despite the small amount of interference that would be incurred. Repeating this for all counties involved might allow each county to be allotted its requested number of frequencies.

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One might use a variety of methodologies to determine the degree of contour overlap (or interference) that is present in a soft constraint assignment set. One individual metric is to use a function of the percent of the coverage contour's area overlapped by the interference contour. Specifically, for coverage contour i , and interference contour j , the soft constraint value entered into the CO constraint matrix would be:

$$CO(i,j) = 1 - (\% \text{ area of coverage contour } i \text{ that is overlapped by interference contour } j)$$

The second term is subtracted from 1 in order to stay consistent with the hard constraint matrix, where "1" means no interference and "0" means interference. In this soft constraint matrix, "1" still means no interference, because there is no overlapping area, and "0" means complete interference everywhere in the county because the entire area is overlapped by the interference contour. This now is a "fuzzy"-valued decision matrix. The closer a "fuzzy" value is to 1, the less interference is encountered by county i from county j . The percent of area overlap may be found by any means, but the example implementation uses a Monte Carlo integration methodology to find the normalized area of the contour intersections.

This is illustrated in Figure 45, in which the coverage contour for one county i is shown in blue, and the interference contour for another county j is shown in green. Because there is overlap present, the hard constraint values, $CO(i,j) = 0$, indicating that county i and j cannot share a frequency. However, there is only a small area where county i and county j interfere. There are 62 sample points in the overlapped area (red) and 871 inside county i (red and black together). So $62/871 = 0.071 = 7.1\%$ of county i is in the overlap area. Thus, the soft constraint matrix, $CO(i,j) = 1 - 0.071 = 0.929$, which is fairly close to 1.

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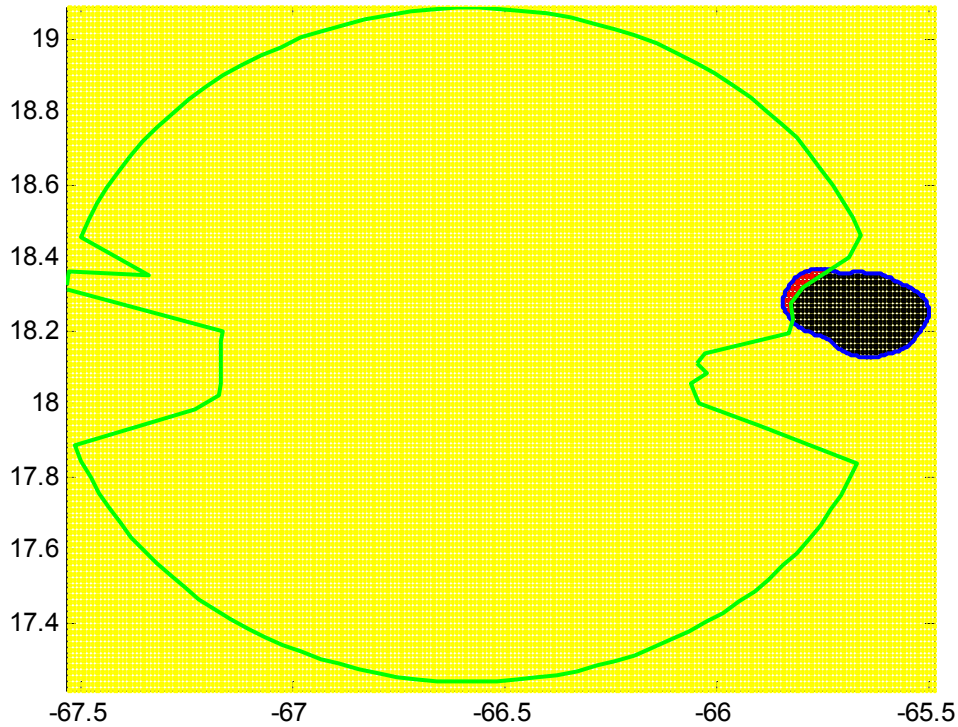


Figure 45, Soft Constraint Overlap through Monte Carlo Integration

On the other hand, Figure 46 shows the coverage contour for county j (blue) and the interference contour for county i (green). There is no intersection, so the soft constraint matrix $CO(j,i)$ is equal to 1, indicating that county j would receive no interference from county i if they shared a frequency. If county i needed more frequencies; it could share frequencies with county j without affecting county j . In that case, county i would be accepting some interference from county j , but county j would still suffer no interference from county i .

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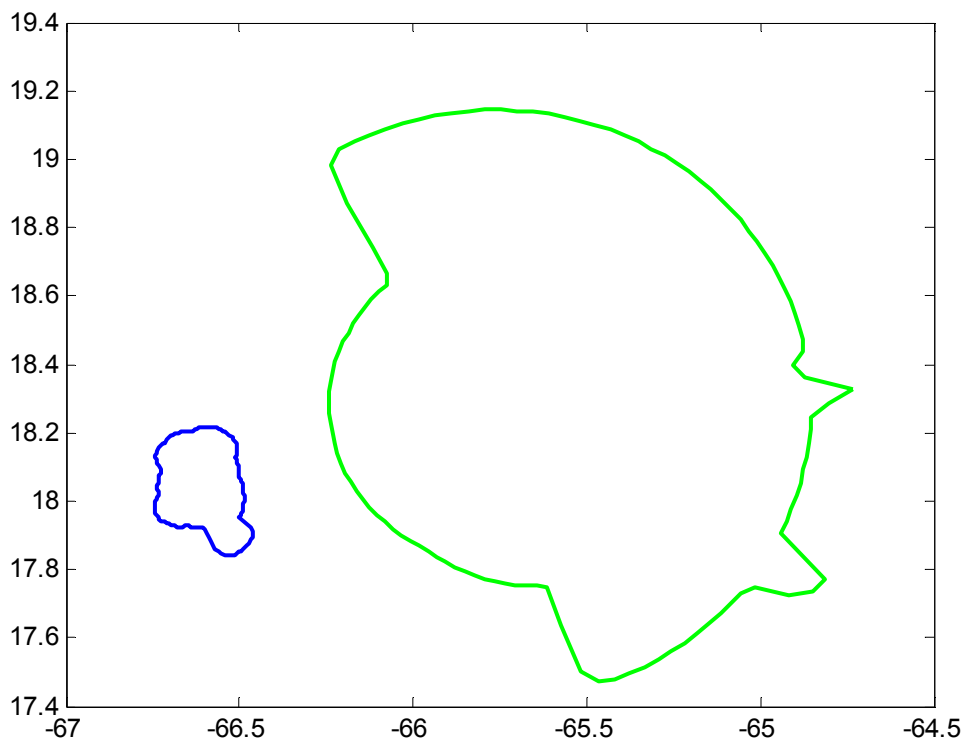


Figure 46, Reverse Case for the Previous Example

The simplest way to use the soft-constraint matrix is to convert it into a binary matrix by converting every entry below a certain threshold to zero and every entry above or at the threshold to one. The threshold may be chosen through iteration or by other means. The next step is to force the matrix to be symmetric by taking the logical “and” of the matrix and its transpose. The symmetric binary matrix can then be used in exactly the same algorithms as the hard-constrained matrix, but now it will represent a soft-constrained problem. For instance, if 1.0 were selected as the threshold, the binary matrix would equal the hard-constrained matrix. If 0.9 were selected as the threshold, the binary matrix would represent permissions allowing co-channel sharing between two counties as long as neither interference contour overlapped more than 10% of the other counties’ service contours. A measure of the total system interference could then be either absorbed into the existing figure-of-merit or optimized separately using a serial approach.

6.5 Algorithm Description

This section gives a detailed heuristic description of the algorithms that were employed under this effort. The algorithms were implemented in MATLAB and may still undergo further improvement and refinement. At this point, some manual intervention was necessary to facilitate

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the final solution that was chosen, but most of the computations are automated. More complex and efficient search space algorithms are under development.

6.5.1 Problem Partition

Because the problem was very computationally intensive, it was advantageous to break it up into separate pieces wherever possible. This can be done if the sites can be partitioned into separate classes such that the assignment problem can be performed independently on each class.

It turns out that there is a unique partition that can be computed from the CO permission matrix. One can say that two sites s_i and s_j directly interfere with each other if $i=j$ or $CO_{i,j}$ is 0. Next, one can say that sites s_i and s_j indirectly interfere with each other if they directly interfere with each other or if there is a third site s_m such that s_i indirectly interferes with s_m and s_m indirectly interferes with s_j . An equivalent definition would be that s_i and s_j indirectly interfere with each other if there is a finite sequence of sites with the first being s_i and the last being s_j such that each site directly interferes with the next.

With this definition, indirect interference is an equivalence relation among sites and thus determines a unique partition of the sites into equivalence classes. Furthermore, assignments made on one class of sites will have no effect on assignments made on any other class. Thus the assignment problem can then be solved independently over each equivalence class of sites.

A MATLAB function was written that computes the partition from the CO permission matrix. For the problem under discussion, there are a total of 3223 sites, and these break down into 4 classes with 3110, 81, 27, and 5 sites, respectively. These correspond to continental U.S., Puerto Rico and Virgin Islands, Alaska, and Hawaii.

This of course is an intuitive result, which could be predicted without resorting to the equivalence class computation. It is also obvious that the bulk of the problem should reside within the continental U.S. However, had the U.S. been partitioned into two or more separate regions, the equivalence class computation would have uncovered the partitions. Furthermore, it was still prudent to separate the problem into the four partitions and solve them independently.

It turns out that the 81 sites in Puerto Rico and Virgin Islands presented a special challenge, due to the size of these Regions (Figure 47), and the high level of constraints within this partition (Figure 48). In fact, only 15% of the county centroids within this partition fall more than 120-km²¹ from one another (also as illustrated in Figure 47). Using hard constraints, it was not possible to achieve the minimum of 4 frequencies within this partition, and only three could be achieved. Since the capacity requirements within this partition were small, it was decided that three frequencies would be acceptable²² for that region.

²¹ A typical co-channel reuse distance.

²² A soft-constraint case was run according to the modifications discussed in subsection 6.4.2. However, the problem within this partition was so over-constrained that the resulting solution would allow too much interference for the minimum of four to be achieved.

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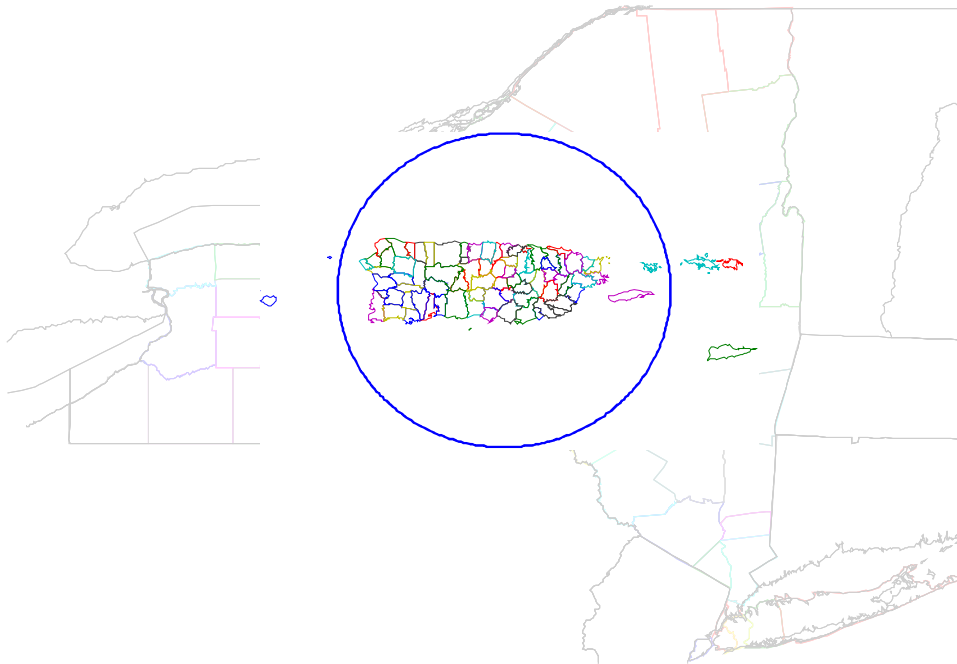


Figure 47, Size of Puerto-Rico and Virgin Islands Partition, Compared to New York State

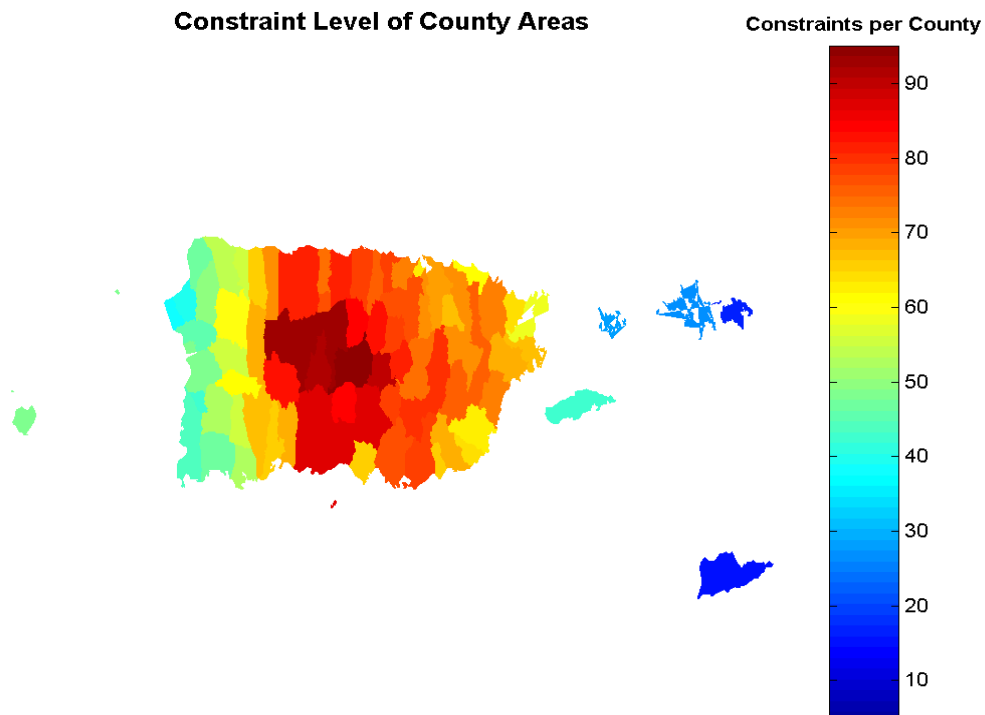


Figure 48, Overall Constraint Levels, Puerto-Rico and Virgin Islands

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6.5.2 Forward-Assignment Algorithm

The basic forward-assignment algorithm begins with an $N \times K$ assignment matrix \underline{A} that is initially all zero-valued, $\underline{A}_o = \underline{0}_{N,N}$, and an $N \times K$ potential matrix \underline{P} that is initially an all-unity matrix, $\underline{P}_o = \underline{1}_{N,N}$. Start by picking a site i and channel j where $P_{i,j}$ is 1 (at the beginning, this could be any site and channel). Next change $A_{i,j}$ to 1 and $P_{i,j}$ to 0. Next change the appropriate \underline{P} values to zero according to the constraints involved from selecting channel j for site i . Now continue this process until \underline{P} is all zero-valued, $\underline{P}_f = \underline{0}_{N,N}$.

This algorithm is guaranteed to find a permissible assignment, because each time an assignment is made all future assignments that are prohibited by the constraints of the assignment just made are eliminated. Further, the assignment obtained is *maximal* in the sense that no further assignments are possible since \underline{P} is all zero.

Of course there is no guarantee that the assignment obtained this way is optimal or even that it is good. Note that the algorithm makes no use of capacity or merit. If the selections are made randomly, any permissible assignment could be obtained this way. Thus one might repeat this algorithm over and over keeping the assignment with the best figure of merit along the way.

The allotment generation here used a forward-assignment algorithm that used both the normalized capacity model and the minimum desired number of frequencies. This algorithm uses merit calculations during the optimization so as to maximize the final figure of merit. The algorithm also has a random aspect to it²³, so that it can be repeated for as long as desired, and will keep the assignment(s) with the best figure of merit.

The basic idea is to always choose a site with the smallest merit; then, among the frequencies available to that site, to choose one that would change the fewest entries in the potential matrix. Any “ties” among sites and frequencies are broken randomly. This process is repeated over and over until the \underline{P} matrix is completely zero-valued.

The forward assignment algorithm was also generalized so that \underline{A} does not have to start at all zeros and \underline{P} does not have to start with all ones. Of course care must be taken to make sure that \underline{P} has zeros where both the initial ones in \underline{A} occur, as well as where the constraints dictate that zeros must be. This generalization allows a portion of the problem to be solved manually or by another algorithm and the rest to be finished by the forward-assignment algorithm. This also allows for the problem to be solved with initial conditions present²⁴.

6.5.3 Site Coloring

Recall that subsection 6.5.1 discussed the notion of interference and the unique partition of the sites so that any two sites in different classes do not interfere. This subsection assumes that one has only one equivalence class as defined in subsection 6.5.1.

²³ For complex discrete combinatorial optimization problems such as this, the most effective algorithms that have been developed to date all utilize structured random search patterns. These methods include Simulated Annealing, Genetic Algorithms, and Ant Colony searches. Problems related to the FAP are minimum map coloring and traveling salesman problems.

²⁴ As the case would be when integrating externally generated pool assignments.

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Now consider the problem of *coloring* the sites in such a way that no two sites with the same color interfere with each other. In graph theory, the coloring problem is to color the nodes of a graph so that no two nodes of the same color are connected and to use the smallest possible number of colors.

Here the nodes are the sites, and the connections are direct interference. Site s_i is connected to site s_j if $CO_{ij} = 0$. Note that, if no two sites of the same color directly interfere, it is also true that no two sites of the same color will interfere indirectly.

Unlike the partition problem of subsection 6.5.1, there is not a unique coloring and the number of colors can vary in different coloring schemes. A MATLAB function was written that computes a coloring from the CO matrix. This function generates different results when the order of the sites is permuted. Trying the program with different random orderings of the sites yields the minimum number of colors obtained for the four different regions mentioned in subsection 6.5.1. These are given in Table 3.

Table 3, Minimum Map Coloring of the Problem Partitions

Region	Number of Sites	Number of Colors
Continental U.S.	3110	26
Puerto Rico and Virgin Islands	81	44
Alaska	27	5
Hawaii	5	3

Note how the number of colors that are required for the Puerto Rico and Virgin Islands partition relative to the number of sites/counties within the partition. This is an indication of how seriously constrained that region is, and why it was not possible to provide four assignments per county there. In fact, a good indicator of the minimum number of frequencies achievable within a partition is to take the total number of frequencies and divide by the minimum number of colors²⁵.

A coloring of sites can be used to compute a very fast partial assignment of frequencies. The idea is to treat each color as a pseudo-site and assign frequencies to them. Then every site with a given color gets the frequencies assigned to that color. This will be a permissible assignment, because sites with the same color do not interfere, and can be given the same frequencies. It is very fast because there are far fewer colors than sites. It is partial because, after a maximal assignment to the colors, it will be possible to make additional assignments to the sites.

In order to apply a forward-assignment algorithm to the colors, one needs to form co-channel and adjacent-channel permission matrices that can be applied to the colors. The co-color matrix must be all zeros, except on the diagonal. The reason is that for any two different colors there must be a pair of sites with those colors that interfere with each other. For the adjacent-color matrix it would be safe to use all zeros; however, for any given pair of colors, if I is the vector of sites

²⁵ This gives $\text{floor}(154/44)=3$ for the PR/VI partition, and $\text{floor}(154/26)=5$ for the Continental U.S. partition. Both are consistent with the minimum number of county blocks assigned within each of these partitions.

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having the first color and J is the vector of sites having the second color, and if $ADJ(I,J)$ are all values of one (1), then the adjacent-color matrix can be 1 for that pair of colors.

A forward-assignment algorithm can now be applied to the colors and a partial assignment on the sites made by assigning to each site all the frequencies that were assigned to the color of that site. Next, a forward-assignment algorithm with the appropriate \underline{A} and \underline{P} can be used to finish the site assignments.

Additional assignments are possible because the partial assignment assumes 1) any two sites of different color directly interfere and 2) any two sites of different color and whose adj value is zero can not be permitted to share adjacent frequencies. Both of these assumptions are too strong for the full assignment problem.

In summary, the color assignment does not cause any constraint violations and does speed up the process considerably. Coloring can be built into the forward-assignment algorithm.

6.5.4 Backward-Refinement Algorithm

The Backward-Refinement Algorithm starts with a completed assignment and tries to modify it so as to improve the figure of merit. Suppose \underline{A} is an assignment and that s_i is a site with merit m_i . Take the vector of all available frequencies and take away those that were assigned to s_i . Next take away those frequencies that violate the single-site constraint for the frequencies that were assigned to s_i . Any frequencies that remain are frequencies that were not assigned to s_i because of site pair constraints.

Let f be one of the remaining frequencies. One could consider assigning f to s_i , but to do that one may have to take f away from some sites and to take frequencies adjacent to f from others. From the \underline{A} , \underline{CO} , and \underline{ADJ} matrices, one can compute exactly which site/frequency pairs would have to be reversed in order to assign f to s_i . This action would change assignment \underline{A} into a different assignment \underline{A}' — both of which would be permissible assignments. The next step is to determine which of the two assignments has the better figure of merit.

It suffices to examine the merit of only the sites that are affected by the change. The site s_i experiences an increase in merit, while the other sites experience a decrease. If the minimum merit of the affected sites under \underline{A}' is greater than the minimum merit of the affected sites under \underline{A} , then the figure of merit of \underline{A}' will be greater than the figure of merit of \underline{A} . In this case, the proposed change would yield an improvement in the figure of merit.

Instead of considering just one of the possible frequencies that can be added to s_i , one could repeat this calculation for each of the possible frequencies. If one (or more) of the possible assignment changes results in an increase in the figure of merit, then implement the change that yields the greatest increase. This process can then be repeated for the same site until no improvement is found. This algorithm will be referred to as *single site improvement*.

The backward-refinement algorithm applied to assignment \underline{A} first sorts the sites according to their merit under \underline{A} , from smallest to largest. The single-site improvement algorithm is then suc-

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cessively applied to each of the sorted sites. The backward-refinement algorithm can be applied repeatedly until there is no change in the assignment.

6.6 Results

After extensive optimization with continual examination of interim results, the final pool allotments were generated. No less than five channel blocks were allotted to each county within the continental United States and Hawaii. As discussed in section 6, certain counties within Puerto Rico and the Virgin Islands were only able to receive three channel blocks, due to the high interference constraints imposed within these areas.

These channels were then delivered to be loaded into the Pre-Coordination Database. Due to the sheer size of the allotment pool, it is not practical to present detailed allotment information within this report; this is available in the database. This section however, provides examples of the results, so that the overall characteristic of the allotment pool can be conveyed.

Figure 49 illustrates the size of the allotment pools for the counties within the continental U.S. (excluding Alaska). Immediately apparent is that none of these counties receives fewer than five channel blocks. Also clear is that the number of allotments is a function of both the capacity model (Figure 15), and the overall constraint levels (Figure 44).

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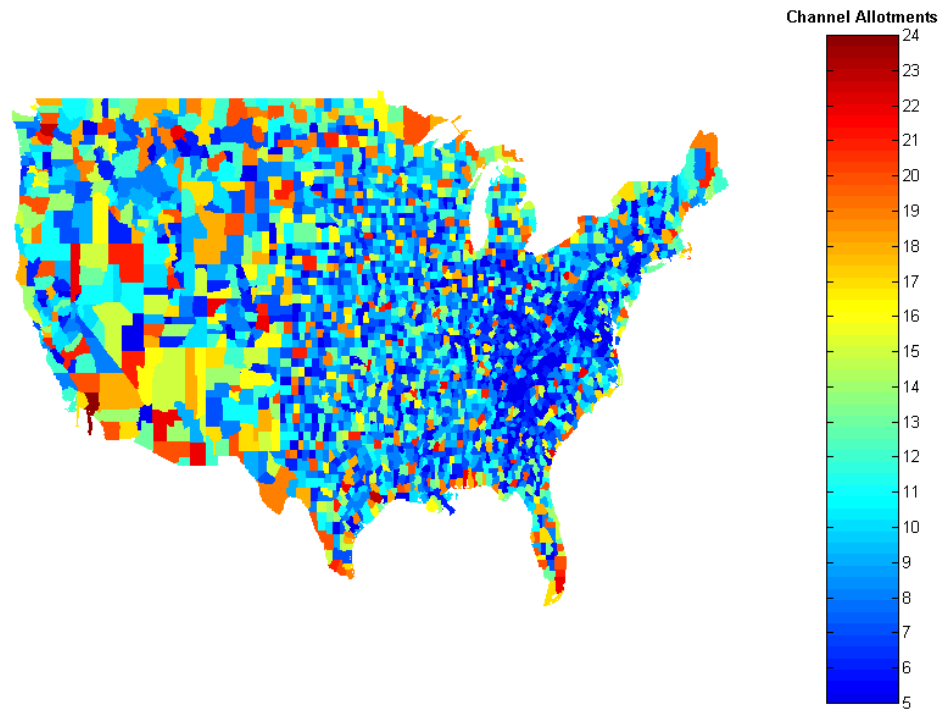


Figure 49, Size of Allotment Pools - Continental US

Figures 50 through 55 are included here in order to show more localized detail of these pool allotment levels in some of the areas with the highest capacity needs, and to also show detail for Puerto-Rico and the Virgin Islands. Note that, except for Figure 50, all of these show no less than five channel blocks allotted per county.

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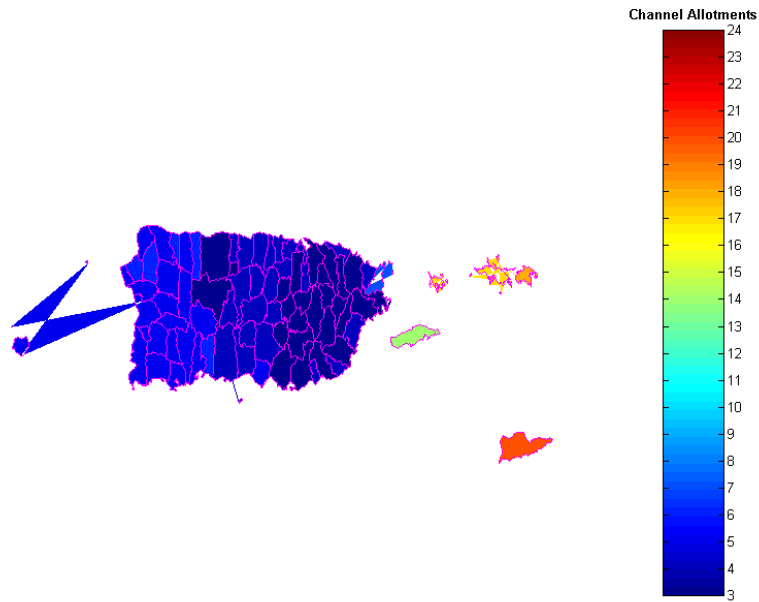


Figure 50, Size of Allotment Pools - Puerto-Rico & U.S. Virgin Islands

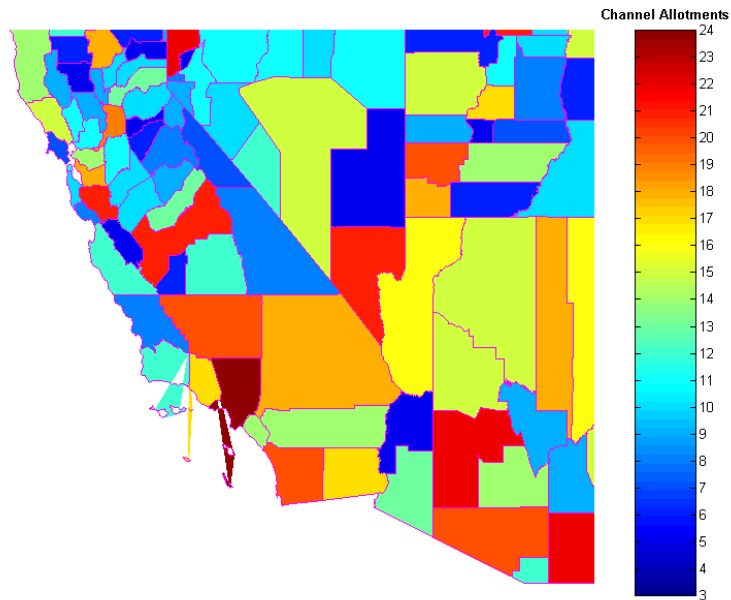


Figure 51, Size of Allotment Pools - Southwestern Continental U.S.

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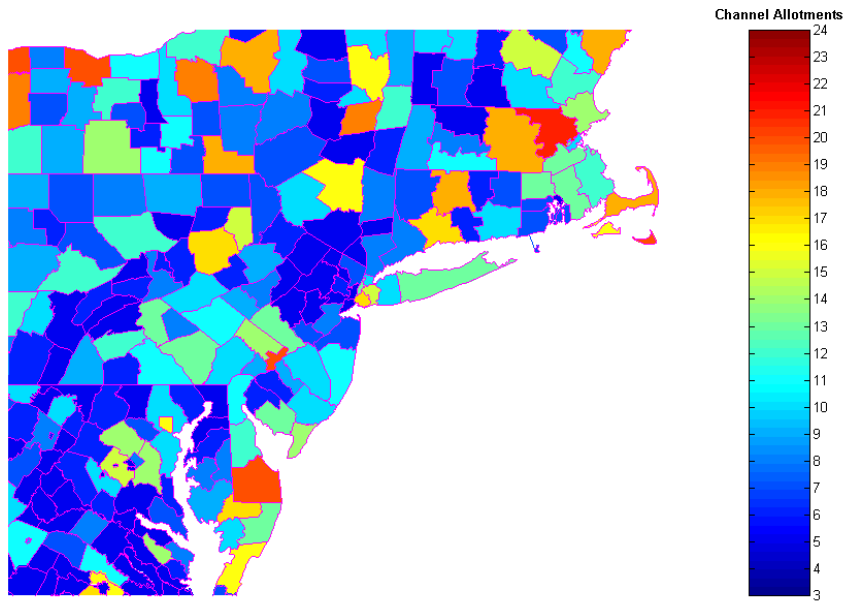


Figure 52, Size of Allotment Pools - Northeastern Continental U.S.

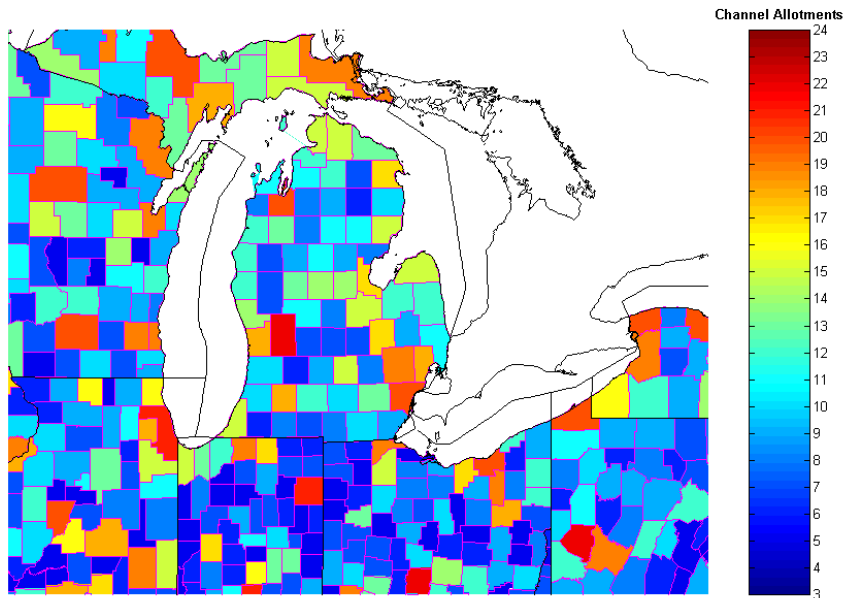


Figure 53, Size of Allotment Pools - North-Central Continental U.S.

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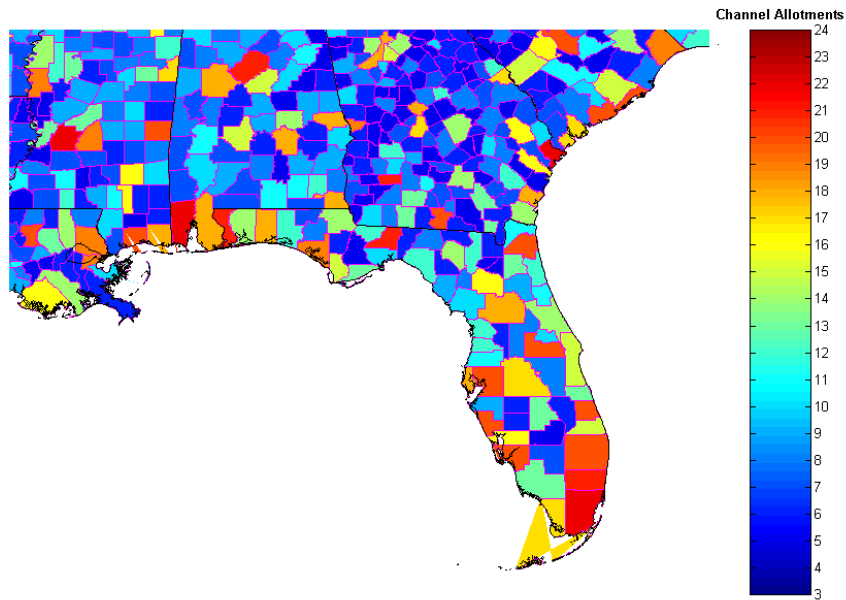


Figure 54, Size of Allotment Pools - Southeastern Continental U.S.

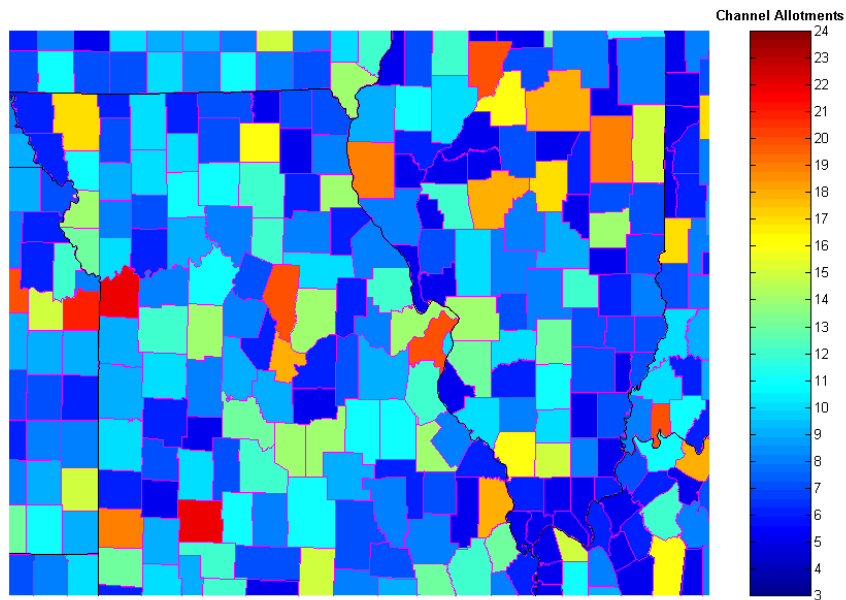


Figure 55, Size of Allotment Pools - St Louis Area

Figure 55 shows the allotment pool size detail for the St Louis (MO) area (centered on Region 24). When comparing this to Figure 16, note that the size of the pool is indeed highly dependent

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upon the capacity model results. Figure 56 is presented here to further illustrate the pool sizes within this Region.

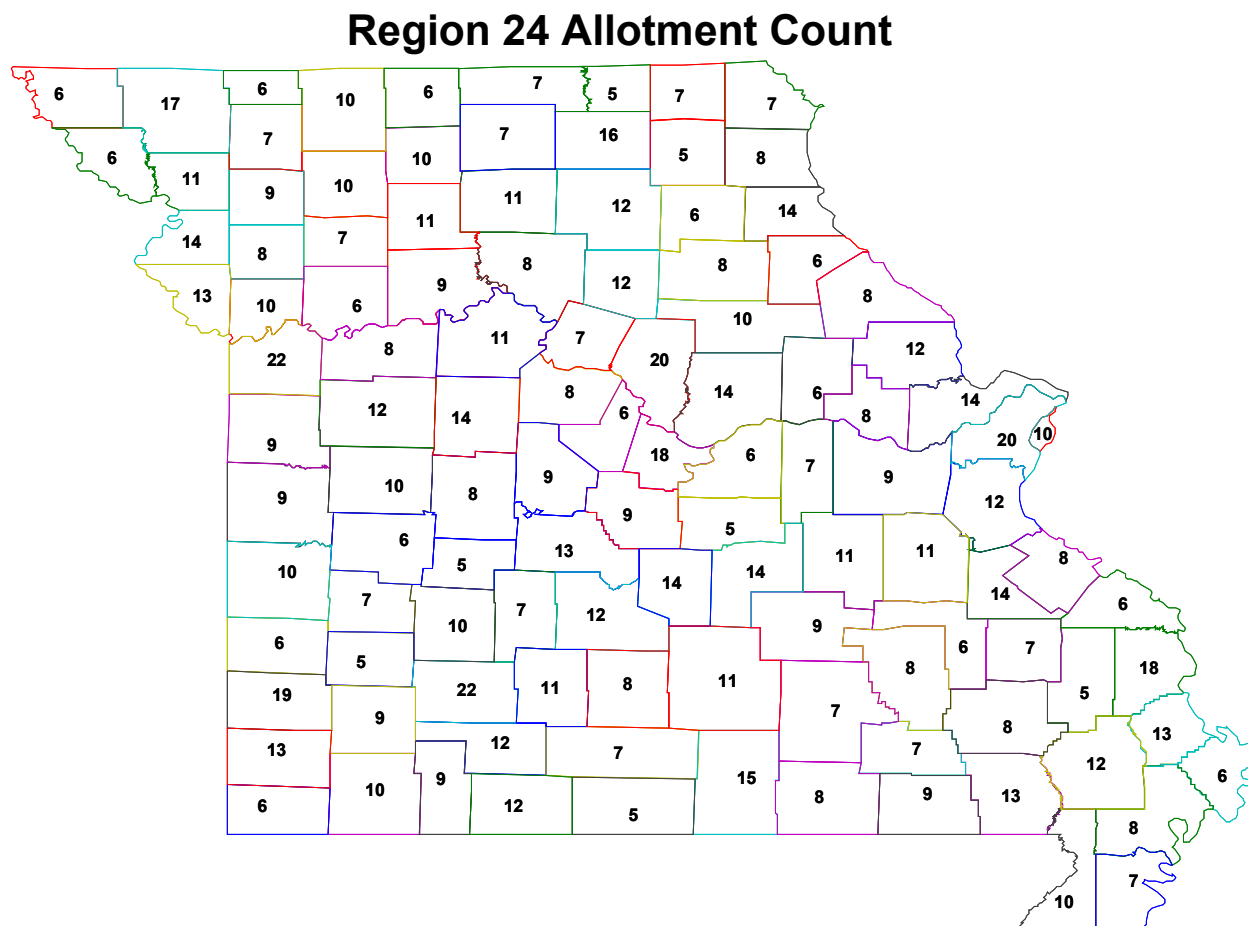


Figure 56, Size of Allotment Pools - Region 24

In general, the pool allotments were generated with the following statistical characteristics:

- Mean Number of Allotments/Channel Blocks per County: 9.82
- Median Number of Allotments/Channel Blocks per County: 8.00
- Mean Number of Reuse (Counties/Channel Block): 205.45
- Median Number of Reuse (Counties/Channel Block): 197.

The distributions of these parameters are further illustrated in Figure 57.

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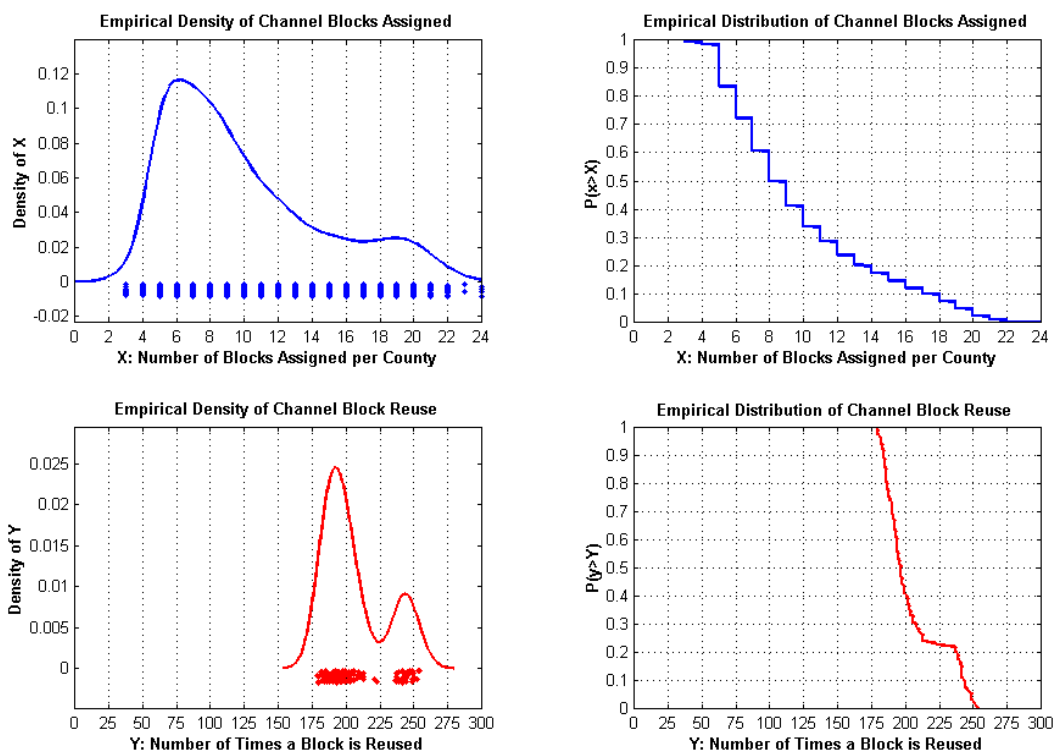


Figure 57, Distributions of Pool Allotments

In closing, look in some more detail at the reuse characteristics of the pool allotments. The most assigned channel block was Block 142 (775.225-775.25 MHz, FCC channel numbers 837-840), which was assigned 254 times within the set of Regions. This is illustrated for the entire Continental U.S. in Figure 58. In this figure, the black-filled areas correspond to interference contours of the co-channel assignments of Block 142, the red-filled areas correspond to the service contours on this block, and the blue-fill represents the assignment of channel blocks adjacent to this block. Figure 59 presents a closer view of this for the Northeast United States.

Figure 60 and Figure 61 show the same information, using contours as opposed to filled-polygons. From these figures, it is easier to see that the co-channel and adjacent-channel allotment constraints are satisfied by the allotment selections. Although under examination it may appear that there are cases where contours do intersect, zooming in on these cases quickly reveals that indeed the constraints were satisfied.

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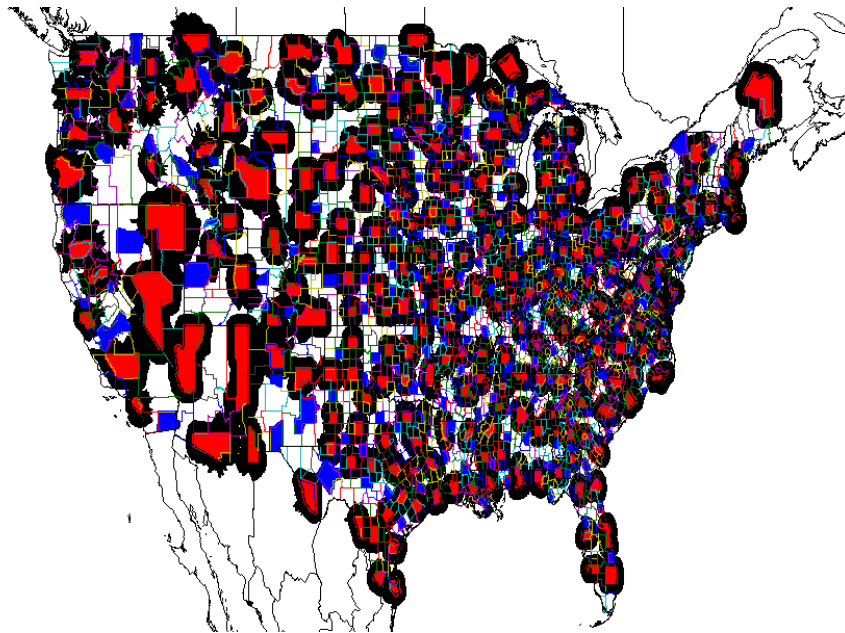


Figure 58, Example of Reuse, Channel Block 142

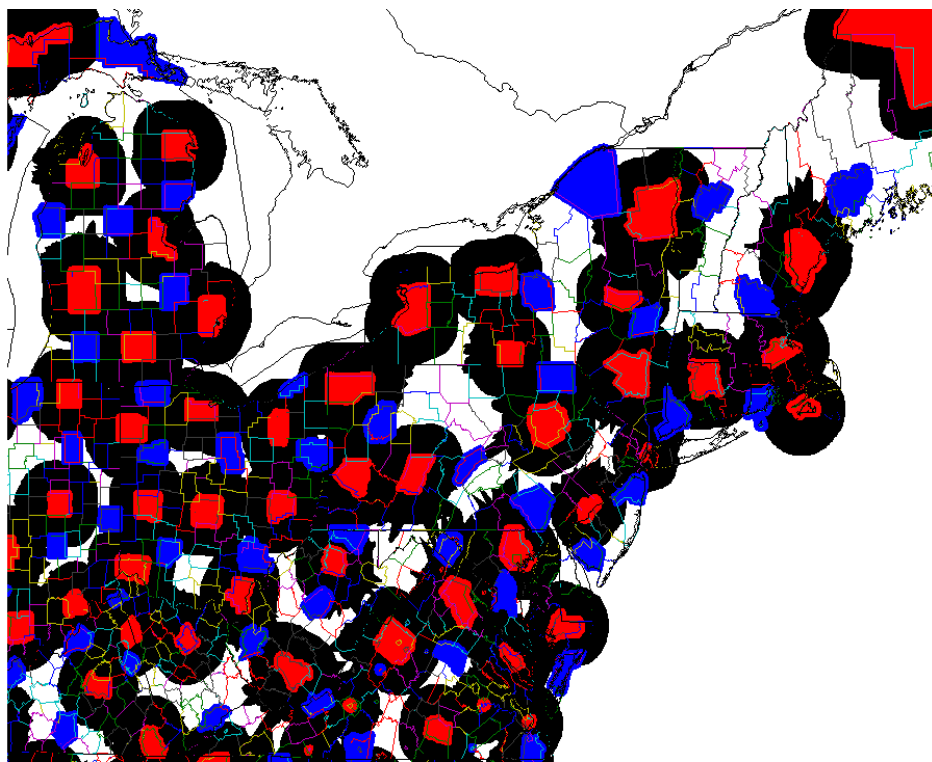


Figure 59, Example of Reuse - NE United States, Channel Block 142

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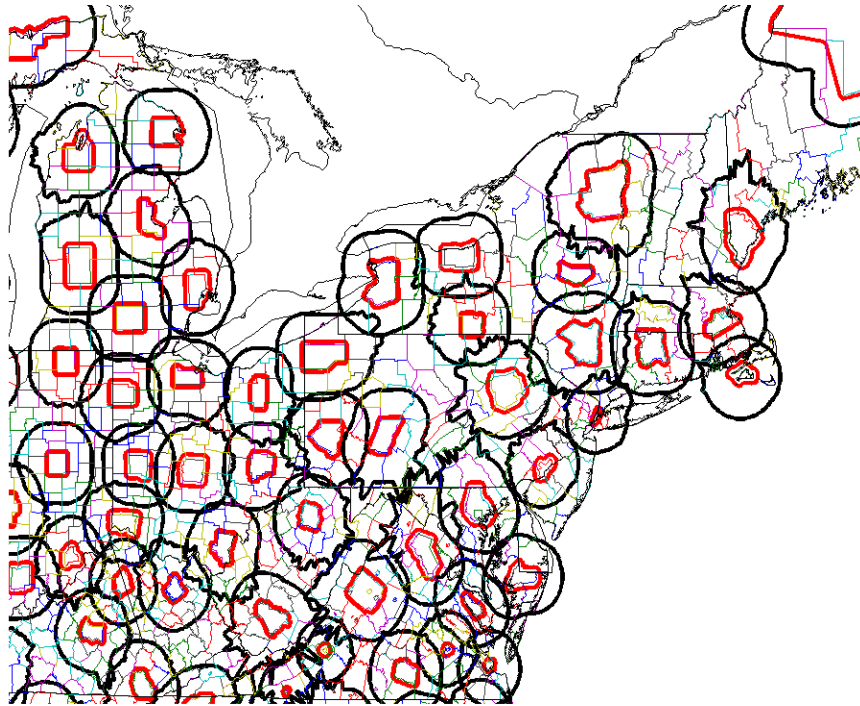


Figure 60, Example of Reuse - NE United States, Channel Block 142 (On-Channel Allotments)

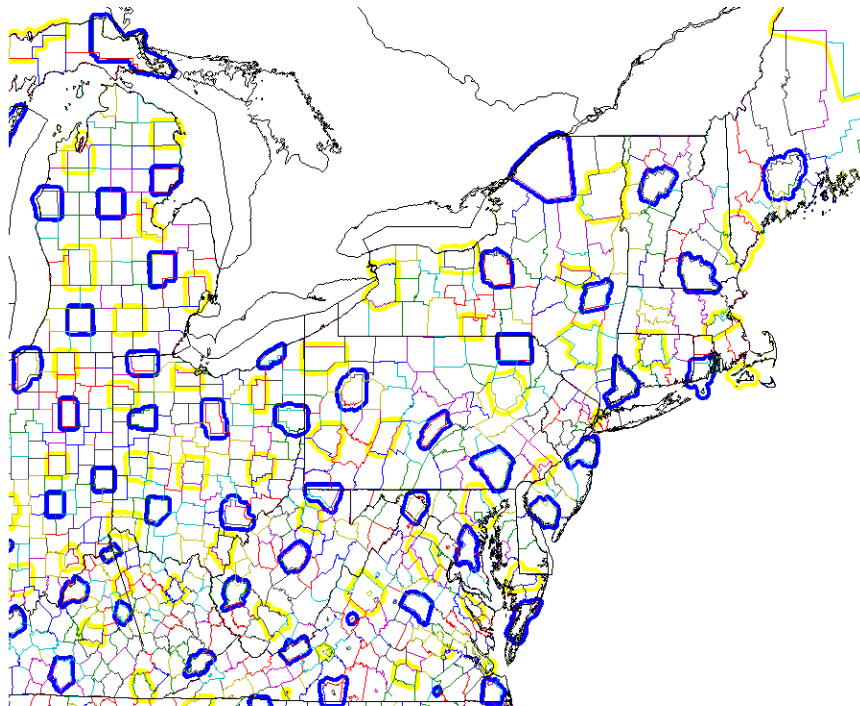


Figure 61, Example of Reuse - NE United States, Channel Block 142 (Adjacent Allotments)

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APPENDIX A - TOP 100 COUNTIES IN TERMS OF CAPACITY NEEDS

State	County	Rank	ERLANG Total
CA	Los Angeles County	1	1945.2539
AZ	Maricopa County	2	982.1424
IL	Cook County	3	964.7525
CA	San Diego County	4	784.3517
CA	San Bernardino County	5	742.6801
TX	Harris County	6	723.2538
FL	Miami-Dade County	7	569.5580
CA	Riverside County	8	542.8023
CA	Orange County	9	538.7155
NV	Clark County	10	506.1446
WA	King County	11	466.8423
NY	Kings County	12	459.4828
TX	Dallas County	13	446.5704
CA	Santa Clara County	14	412.6374
MI	Wayne County	15	394.2804
FL	Broward County	16	393.3232
NY	Queens County	17	384.1817
AZ	Pima County	18	360.0404
TX	Bexar County	19	354.2443
NY	New York County	20	330.4446
NY	Suffolk County	21	326.4599
FL	Palm Beach County	22	324.7740
TX	Tarrant County	23	324.0926
MA	Middlesex County	24	321.5413
CA	Fresno County	25	312.0421
CA	Alameda County	26	307.4261
CA	Sacramento County	27	303.2331
CA	Kern County	28	290.8069
MI	Oakland County	29	287.6664
PA	Allegheny County	30	282.5669
OH	Cuyahoga County	31	270.9669
FL	Hillsborough County	32	261.3035
PA	Philadelphia County	33	254.5321
NY	Erie County	34	250.4967

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State	County	Rank	ERLANG Total
NY	Nassau County	35	244.9168
NY	Bronx County	36	244.1546
MN	Hennepin County	37	236.2994
FL	Orange County	38	232.8843
CA	Contra Costa County	39	231.7558
CA	Ventura County	40	231.1256
OH	Franklin County	41	226.9923
TN	Shelby County	42	225.8072
UT	Salt Lake County	43	225.1173
MA	Worcester County	44	221.6823
TX	Travis County	45	218.1124
CT	Hartford County	46	216.4179
MO	St. Louis County	47	215.2551
WA	Pierce County	48	214.0883
CT	Fairfield County	49	210.2963
HI	Honolulu County	50	206.1259
FL	Duval County	51	201.6913
WA	Snohomish County	52	199.4951
CT	New Haven County	53	198.9300
VA	Fairfax County	54	196.2572
NY	Westchester County	55	192.6803
MD	Montgomery County	56	192.2715
TX	El Paso County	57	189.3986
AL	Jefferson County	58	188.7173
GA	Fulton County	59	188.2218
MD	Baltimore City	60	187.3240
NY	Monroe County	61	187.0622
MD	Prince George's County	62	180.5228
IL	Du Page County	63	179.4323
TX	Hidalgo County	64	178.9689
IN	Marion County	65	178.8156
MI	Macomb County	66	177.9171
OH	Hamilton County	67	177.5111
FL	Pinellas County	68	176.8010
CO	El Paso County	69	176.6892
WI	Milwaukee County	70	176.1079
CA	San Joaquin County	71	173.4087
OK	Oklahoma County	72	173.3827

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State	County	Rank	ERLANG Totals
PA	Montgomery County	73	172.6583
NC	Wake County	74	171.1967
MA	Essex County	75	170.8672
NC	Mecklenburg County	76	169.7012
MO	Jackson County	77	167.4485
NJ	Bergen County	78	166.2747
NM	Bernalillo County	79	165.5852
NV	Washoe County	80	165.4956
CA	Tulare County	81	164.2689
FL	Polk County	82	163.2868
CA	San Mateo County	83	161.9588
CA	Monterey County	84	160.6211
MI	Kent County	85	160.0577
PA	Bucks County	86	155.3802
OR	Multnomah County	87	153.2758
CA	Santa Barbara County	88	152.8802
IL	Lake County	89	152.3920
NJ	Middlesex County	90	152.2399
KY	Jefferson County	91	151.5599
NJ	Monmouth County	92	150.8905
CA	Sonoma County	93	150.8683
MA	Norfolk County	94	147.2131
OR	Lane County	95	146.7630
CO	Jefferson County	96	146.4042
OK	Tulsa County	97	146.3550
CA	Stanislaus County	98	146.1898
RI	Providence County	99	144.7121
TN	Davidson County	100	144.5467

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APPENDIX B - POPULATION, AREA, AND CAPACITY MODEL DATA